## MARS Tasks | Course 1

Page

|  | Name of MARS Task | Mear | Math Strand | Notes |  |
| :---: | :--- | :--- | :--- | :--- | :---: |
| $*$ | Vacuum Cleaning | 2003 | GM | Draw, identify regions in context |  |
| $*$ | Snakes | 2003 | DA | Interpret scatter plots, identify values |  |
| $*$ | Crisscross Numbers | 2003 | PFA | Use algebra to explain number patterns |  |
| $*$ | Conference Tables | 2003 | PFA, GM | Find/extend pattern in geometric context |  |
| $*$ | Number Towers | 2003 | PFA | Form/solve equations from number pattern |  |
|  |  |  |  |  |  |
| $*$ | Square Patterns | 2004 | PFA | Work with patterns, write formula |  |
| $*$ | Population | 2004 | DA, PFA | Interpret scatter plot, perform operations |  |
| $*$ | From 2 to 3 Dimensions | 2004 | GM | Imagine 3-D from 2-D net, compare |  |
| $*$ | Graphs | 2004 | PFA | Relate equations, descriptions, graphs |  |
| $*$ | Fibonacci Sequences | 2004 | PFA | Use algebra to solve number problems |  |


| 2 | Magic Squares | 2005 | PFA | Calculate values using algebraic notation |
| ---: | :--- | :--- | :--- | :--- |
| 5 | Vacations | 2005 | PFA | Analyze relationships w/graphs, algebra |
| 9 | Multiples of Three | 2005 | PFA | Test statement, find examples, justify |
| 13 | Scatter Diagram | 2005 | DA | Discuss, understand scatter plot |
| 17 | Fraction Sequences | 2005 | PFA | Extend sequence with fractions, decimals |


| 21 | Swimming Pool | 2006 | GM, PFA | Work w/trapezoids, rates, and time graphs |
| :--- | :--- | :--- | :--- | :--- |
| 25 | Odd Sums | 2006 | PFA | Word w/odd, even \& consecutive numbers |
| 29 | Patchwork Quilt | 2006 | GM, PFA | Extend pattern to express rule using algebra |
| 33 | Printing Tickets | 2006 | PFA | Compare prices using graphs, formulas |
| 37 | Graphs | 2006 | PFA | Relate line graphs to equations |


| 41 | Graphs | 2007 | PFA | Work with functions, graphs, equations |
| ---: | :--- | :--- | :--- | :--- |
| 45 | House Prices | 2007 | PFA | Graphs, formulas in real context |
| 49 | Ash's Puzzle | 2007 | PFA | Find numbers that obey rules, find rules |
| 53 | How Old Are They? | 2007 | PFA | Form/solve equation to solve age problem |
| 57 | Two Solutions | 2007 | PFA | Find solutions to equations, inequalities |


| 61 | Expressions | 2008 | PFA | Algebraic expressions for shapes |
| ---: | :--- | :--- | :--- | :--- |
| 64 | Buying Chips and Candy | 2008 | PFA | Form/solve linear equations in context |
| 68 | Sorting Functions | 2008 | PFA | Find/explain graphs, equations, tables, rules |
| 72 | Sidewalk Patterns | 2008 | PFA | Work with patterns, sequences |
| 76 | Functions | 2008 | PFA | Work with linear/non-linear functions |


| 80 | Soup and Beans | 2009 | PFA | Make equation to solve problem |
| ---: | :--- | :--- | :--- | :--- |
| 83 | Quadratic | 2009 | PFA | Work with quadratic function |
| 87 | Circles and Spheres | 2009 | PFA | Compare functions: length, area, volume |
| 91 | Words and Equations | 2009 | PFA | Write equations to match situations |
| 99 | Coffee | 2009 | PFA | Use chart to solve simultaneous equations |

NP=Number Properties
$\mathrm{NO}=$ Number Operations
PFA=Patterns Functions Algebra GM=Geometry \& Measurement DA=Data Analysis

* Tasks from 2003 and 2004 are not included in this packet due to copyright restrictions. However, if you click on the name of the task, you can access it via the Noyce Foundation website. Tasks from 2005 to 2009 are available here with permission from the Mathematics Assessment Resource Service (MARS).

Course 1 Task 1 Magic Squares

| Student Task | Use symbolic algebraic notation to calculate values in "magic" squares where each row, column and diagonal adds to the same number. |
| :---: | :---: |
| Core Idea <br> 3 <br> Algebraic <br> Properties and <br> Representations | Represent and analyze mathematical situations and structures using algebraic symbols. <br> - Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations <br> - Write equivalent forms of equations, inequalities and systems of equations and solve them <br> - Use symbolic algebra to represent and explain mathematical relationships |
| Core Idea <br> 2 <br> Mathematical <br> Reasoning | Employ forms of mathematical reasoning and proof appropriate to the solution of the problem, including deductive and inductive reasoning, making and testing conjectures and using counterexamples and indirect proof. <br> - Show mathematical reasoning in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models |

## Magic Squares

This problem gives you the chance to:

- work with magic squares, calculating cell values
- understand simple algebraic notation

| 6 | 1 | 8 |
| :--- | :--- | :--- |
| 7 | 5 | 3 |
| 2 | 9 | 4 |

In this square, adding the numbers in each row, each column and each diagonal gives the same result.

For such a square to be a "magic square" all nine numbers must be different.

1. Find the sum of each row, each column and each diagonal for this magic square. $\qquad$

| $x+z$ | $x-y-z$ | $x+y$ |
| :---: | :---: | :---: |
| $x+y-z$ | $x$ | $x-y+z$ |
| $x-y$ | $x+y+z$ | $x-z$ |

This is the general form of a magic square, in which $\mathrm{x}, \mathrm{y}$ and z represent numbers.
2. Find the sum of each row, each column and each diagonal for this square.
$\qquad$

Here is a partially completed magic square.
3. Use algebra to complete this magic square.

4. Find the sum of each row, each column and each diagonal for the completed square. $\qquad$

| Magic Squares |  |  |  | Rubric |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - work with magic squares, calculating cell values <br> - understand simple algebraic notation <br> Based on these, credit for specific aspects of performance should be assigned as follows |  |  |  | points | section points |
| 1. Gives correct answer: $\mathbf{1 5}$ |  |  |  | 1 | 1 |
| 2. Gives correct answer: 3x |  |  |  | 2 | 2 |
| 3. All five values correct. <br> Partial credit 4 or 3 values correct <br> Some evidence of the correct use of algebra. | 12 11 4 | 1 9 17 | $\begin{array}{\|c\|} \hline 14 \\ \hline 7 \\ \hline 6 \\ \hline \end{array}$ | (1) $2$ | 4 |
| 4. Gives correct answer: 27 |  |  |  | 1 | 1 |
| Total Points |  |  |  |  | 8 |

Course 1 Task 2
Vacations

| Student Task | Match graphic displays to the written descriptions of how some students are paying for their summer vacations. Write a formula that describes each of the matched relationships and then write a possible description for a new vacation saving formula. |
| :---: | :---: |
| $\begin{array}{\|l} \hline \text { Core Idea } \\ \mathbf{1} \\ \text { Functions and } \\ \text { Relations } \\ \hline \end{array}$ | Understand patterns, relations, and functions. <br> - Generalize patterns using explicitly defined functions <br> - Understand relations and functions and select, convert flexibly among, and use various representations for them <br> - Analyze functions of one variable by investigating local and global behavior, including slopes as rates of change, intercepts and zeros |
| Core Idea <br> 3 <br> Algebraic <br> Properties and Representations | Represent and analyze mathematical situations and structures using algebraic symbols. <br> - Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations <br> - Use symbolic algebra to represent and explain mathematical relationships <br> - Use symbolic expressions to represent relationships arising from various contexts <br> - Approximate and interpret rates of change from graphic and numeric data |
| Core Idea <br> 2 <br> Mathematical <br> Reasoning | Employ forms of mathematical reasoning and proof appropriate to the solution of the problem, including deductive and inductive reasoning, making and testing conjectures and using counterexamples and indirect proof. <br> - Use induction to make conjectures and use deductive reasoning to prove conclusions <br> - Draw reasonable conclusions about a situation being modeled |

## Vacations

This problem gives you the chance to:

- analyze relationships using graphs and algebra

Here is some information about how some students are paying for their summer vacations.
Carla: Her mom gave her $\$ 100$ in January and Carla has saved $\$ 25$ every month since, starting in February.
Arnie: Arnie put $\$ 150$ in his piggy bank in January.
Sue: Sue booked her vacation in January. She had $\$ 250$ in her piggy bank. Starting in February, she is paying $\$ 50$ each month to the travel company.

Ben: Starting in February, Ben saves $\$ 30$ every month.
Here are some graphs illustrating these situations.

1. Match each person with a graph and explain how you decided.


Name: $\qquad$
Reason: $\qquad$
$\qquad$
$\qquad$


Name: $\qquad$
Reason: $\qquad$


Name: $\qquad$
Reason: $\qquad$
$\qquad$
$\qquad$
2. In these equations, $\$ A$ is the amount of money and $n$ is the number of months since January.

$$
\begin{aligned}
& A=250-50 n \\
& A=30 n \\
& A=150
\end{aligned}
$$

a. Find the person for each of these equations.
b. Write a formula for the fourth person.

Carla $\qquad$
Arnie $\qquad$
Sue $\qquad$
Ben $\qquad$
3. Write a possible description for this formula: $A=50 n+150$
$\qquad$
$\qquad$

| Vacation s | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - analyze relationships using graphs and algebra <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. For each graph: <br> Gives the correct name and identifies one or more features of the graph or situation that distinguishes it from the others, such as the intercept or slope. <br> Sue: graph goes downhill, showing that the amount of money in her piggy bank is decreasing. Intercept $\$ 250$ : Slope $<0$ or $-\$ 50$ <br> Arnie: graph is horizontal, showing the amount of money in his piggy bank stays the same. Intercept $\$ 150$ : Slope constant or 0 <br> Ben: graph starts at 0 and goes up in steady steps. Intercept \$0: Slope $>0$ \$30 <br> Carla: graph starts above 0 and goes up in steady steps. Intercept $\$ 100$ : Slope $>0$ or $\$ 25$ | 1 <br> 1 <br> 1 | 4 |
| 2. a. Carla: none of the given equations <br> Arnie: $A=150$ <br> Sue: $A=250-50 n$ <br> Ben: $A=30 n$ <br> All 3 correct <br> Partial credit <br> 2 correct <br> b. G ives a correct formula for Carla: $\boldsymbol{A}=\mathbf{1 0 0 + 2 5 n}$ | 2 <br> (1) <br> 1 | 3 |
| 3. Gives a correct description such as: <br> Student starts with $\$ 150$ and saves $\$ 50$ a month. | 1 | 1 |
| Total Points |  | 8 |

## Course 1 Task 3 Multiples of Three

| Student Task | Given a statement regarding multiples of three, test it to see if it is true, find examples that match the statement and explain and justify conclusions. |
| :---: | :---: |
| Core Idea 3 <br> Algebraic <br> Properties and Representations | Represent and analyze mathematical situations and structures using algebraic symbols. <br> - Compare and contrast the properties of numbers and number systems including real numbers <br> - Use symbolic algebra to represent and explain mathematical relationships <br> - Use symbolic expressions to represent relationships arising from various contexts |
| Core Idea <br> 2 <br> Mathematical <br> Reasoning | Employ forms of mathematical reasoning and proof appropriate to the solution of the problem, including deductive and inductive reasoning, making and testing conjectures and using counterexamples and indirect proof. <br> - Explain the logic inherent in a solution process <br> - Use induction to make conjectures and use deductive reasoning to prove conclusions <br> - Draw reasonable conclusions about a situation being modeled |

## Multiples of Three

This problem gives you the chance to:

- test statements to see if they are true
- find examples to match a description
- explain and justify your conclusions


## If a number is a multiple of three, its digits add up to a multiple of three.

For example, 15 is a multiple of three $(15=3 \times 5)$ and $1+5=6$, which is a multiple of three.

Also, the number 255 is a multiple of three $(255=3 \times 85)$ and $2+5+5=12$, which is a multiple of three.

1. Use the above rule to test whether 4721 is a multiple of three or not and explain how you figured it out.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
2. Use the above rule to find a 5-digit multiple of three and explain how you know you are correct.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
3. Zara says, "If you add two multiples of three you always get another multiple of three."

Is Zara correct? $\qquad$
Explain how you decided.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
4. Phil says, "If you add two multiples of three you always get a multiple of six."

Is Phil correct? $\qquad$
Explain how you decided.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

| Multiples of Three | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - test statements to see if they are true <br> - find examples to match a description <br> - explain and justify your conclusions <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answer: $\mathbf{4 7 2 1}$ is not a multiple of $\mathbf{3}$ <br> Gives an explanation such as: $4+7+2+1=14$ <br> 14 is not a multiple of 3 , so 4721 is not a multiple of 3 | $\begin{aligned} & 1 \\ & 1 \\ & 1 \end{aligned}$ | 3 |
| 2. Writes down a five digit number such as 21,111 <br> Adds the digits: $2+1+1+1+1=6$ <br> States that since 6 is a multiple of 3 <br> 21,111 is a multiple of 3 <br> $(21,111=3 \times 7,037)$ |  | 2 |
| 3. Gives correct answer: Yes supported by a correct explanation such as: If two numbers are multiples of 3 , then they can be written in the form $3 x$ and $3 y$ (where $x$ and $y$ are integers) $3 x+3 y=3(x+y)$, which is a multiple of 3 | 2 | 2 |
| 4. Gives correct answer: No with one counterexample such as: $6+15=21$ | 1 | 1 |
| Total Points |  | 8 |

[^0]| Student <br> Task | Explain the information presented in a scatter plot of students’ scores <br> on two tests. Evaluate statements made about the relationships <br> found from the data and revise the statements if necessary. |
| :--- | :--- |
| Core Idea <br> $\mathbf{5}$ <br> Data Analysis | Select and use appropriate statistical methods to analyze data <br> and understand and apply basic concepts of probability. <br> - <br> Understand the relationship between two sets of data <br> (bivariate) and describe trends and shape of the plot including <br> correlations (positive, negative, or no) and lines of best fit <br> Make inferences based on the data and evaluate the validity <br> of conclusions drawn |
| Core Idea <br> $\mathbf{Z}$ <br> Mathematical <br> Reasoning | Employ forms of mathematical reasoning and proof appropriate <br> to the solution of the problem, including deductive and inductive <br> reasoning, making and testing conjectures and using <br> counterexamples and indirect proof. <br> - Use induction to make conjectures and use deductive <br> reasoning to prove conclusions |
| Draw reasonable conclusions about a situation being modeled |  |

## Scatter Diagram

This problem gives you the chance to:

- discuss and understand a scatter plot of real data

A group of 66 students took two tests; Test A and Test B.
In the scatter diagram, each square represents one student and shows the scores that student got in the two tests.

Scores in Test A and Test B


1. The mean score for Test A was 19 and the mean score for Test B was 16 .

Plot a point to show this on the scatter diagram.
2. Draw a line of best fit on the scatter diagram.

How can a line of best fit be used?
$\qquad$
$\qquad$
3. Here are five statements about the scores shown on the scatter diagram.

If a statement is true check $(\sqrt{ })$ it.
If it is not true, write a correct statement.

| Statement | Check (V) or write correct statement |
| :--- | :--- |
| The scatter diagram shows positive <br> correlation between the scores on Test A <br> and the scores on Test B. |  |
| The lowest score on Test A is lower than <br> the lowest score for Test B. |  |
| The range of scores on Test B is 25. |  |
| The student with the highest score on <br> Test A also has the highest score on <br> Test B. |  |
| The biggest difference between a <br> student's scores on the two tests is 5. |  |


| Scatter Diagram | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - discuss and understand a scatter plot of real data <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Point correctly plotted | 1 | 1 |
| 2. Draws a line that best fits the data. <br> Gives a correct statement such as: <br> A line of best fit can be used to estimate a students' score in one test if you know their score in the other. | 1 <br> 1 | 2 |
| 3. Correctly completes the table such as: $\sqrt{ }$ <br> No. The lowest score on Test A (7) is greater than the lowest score on Test B (3). <br> $\sqrt{ }$ <br> No. The student with the highest score on Test A (30) does not have the highest score on Test B (28). <br> No. The biggest difference is more than 5 (accept 14). |  | 5 |
| Total Points |  | 8 |

## Course 1 Task 5 Fraction Sequences

| Student <br> Task | Extend a sequence of fractions and compare the values. Make <br> conjectures about the patterns in the values of the terms as well as <br> their equivalent decimal values. |
| :--- | :--- |
| Core Idea <br> $\mathbf{1}$ <br> Functions <br> and Relations | Understand patterns, relations, and functions. <br> - <br> - Generalize patterns using explicitly defined functions <br> Understand relations and functions and select, convert <br> flexibly among, and use various representations for them |
| Core Idea <br> $\mathbf{2}$ <br> Mathematical <br> Reasoning | Employ forms of mathematical reasoning and proof appropriate <br> to the solution of the problem, including deductive and inductive <br> reasoning, making and testing conjectures and using <br> counterexamples and indirect proof. <br> - Use induction to make conjectures and use deductive <br> reasoning to prove conclusions |

## Fraction Sequences

This problem gives you the chance to:

- extend a given sequence of fractions
- calculate and compare decimal values

Bob writes a sequence of fractions.
In his sequence the next term after $\frac{x}{y}$ is $\frac{y+x}{y+2 x}$
For example, the term after $\frac{2}{3}$ is $\frac{3+2}{3+4}=\frac{5}{7}$
Bob begins to write his sequence in a table.

|  |  | Decimal value |
| :--- | :---: | :--- |
| term \# 1 | $\frac{2}{3}$ | 0.66666666 |
| term \# 2 | $\frac{5}{7}$ | 0.7142857 |
| term \# 3 | $\frac{12}{17}$ | 0.705882 |
| term \# 4 |  |  |
| term \# 5 |  |  |
| term \# 6 |  |  |

1. Calculate term \# 4, term \#5, term \# 6 in Bob's fraction sequence, and write them in the table.
2. What do you notice about the decimal values of these terms?
$\qquad$
$\qquad$

Anna writes a different sequence of fractions.
In her sequence the next term after $\frac{x}{y}$ is $\frac{y-x}{y+x}$
For example, the term after $\frac{2}{3}$ is $\frac{3-2}{3+2}=\frac{1}{5}$
Anna begins to write her sequence in a table.

|  |  | Decimal value |
| :--- | :--- | :--- |
| term \# 1 | $\frac{2}{3}$ | 0.66666666 |
| term \# 2 | $\frac{1}{5}$ | 0.2 |
| term \# 3 |  |  |
| term \# 4 |  |  |
| term \#5 |  |  |
| term \#6 |  |  |

3. Calculate term \# 3, term \# 4, term \# 5, term \# 6 in Anna's fraction sequence and write them in the table.
4. What do you notice about the values of these terms?
$\qquad$
$\qquad$

## Fraction Sequences

The core elements of performance required by this task are:

- extend a given sequence of fractions
- calculate and compare decimal values

Based on these, credit for specific aspects of performance should be assigned as follows

1. Correctly completes table:

|  |  | Decimal value |
| :--- | :---: | :--- |
| term \# 1 | $\frac{2}{3}$ | 0.66666666 |
| term \# 2 | $\frac{5}{7}$ | 0.7142857 |
| term \# 3 | $\frac{12}{17}$ | 0.705882 |
| term \# 4 | $\frac{29}{41}$ | 0.707317 |
| term \# 5 | $\frac{70}{99}$ | 0.707070 |
| term \# 6 | $\frac{169}{239}$ | 0.7071129 |

6 correct values
3
Partial credit
5 or 4 correct values
3 or 2 correct values
2. Gives a correct statement such as:
from term \#2 on, the values are around 0.7 .
Accept: the values increase then decrease.
3. Correctly completes table:

|  |  | Decimal value |
| :--- | :--- | :--- |
| term \# 1 | $\frac{2}{3}$ | 0.66666666 |
| term \# 2 | $\frac{1}{5}$ | 0.2 |
| term \# 3 | $\frac{2}{3}$ | 0.66666666 |
| term \# 4 | $\frac{1}{5}$ | 0.2 |
| term \# 5 | $\frac{2}{3}$ | 0.66666666 |
| term \# 6 | $\frac{1}{5}$ | 0.2 |

8 correct values
Partial credit
7, 6 or 5 correct values
4,3 or 2 correct values
4. Gives a correct statement such as:
values oscillate between 0.2 and 0.666
Total Points
(2)

## Algebra

## Course One/Algebra

## Task 1

Swimming Pool

| Student Task | Work with trapezoids, volume, rates and time graphs in the context of <br> a swimming pool. |
| :--- | :--- |
| Core Idea 4 <br>  <br> Measurement | Understand measurable attributes of objects; and understand the <br> units, systems, and process of measurement. |
| Core Idea 3 <br> Alg. Properties <br> $\boldsymbol{\&}$ | Approximate and interpret rates of change, from graphic and <br> numeric data. |
| Representations | - Analyze functions of one variable by investigating local and |
| Core Idea 1 <br> Functions and <br> Relations | global behavior, including slopes as rates of change, intercepts <br> and zeros. |

Based on teacher observation, this is what algebra students knew and were able to do:

- Students were able to convert from seconds to hours, but were unsure what to do with the decimal

Areas of difficulty for algebra students:

- Finding volume of an unfamiliar shape
- Composing/ decomposing a shape into familiar parts
- Confusing a state rate of water flow with a steady rise in the depth of the pool
- Confusing the shape of the pool with the shape of the graph
- Not recognizing that after 5 feet the depth would increase at a steady rate


## Swimming Pool

This problem gives you the chance to:

- work with trapezoids, rates and time graphs in a real context

This diagram shows a swimming pool.
The top of the swimming pool is a rectangle measuring 30 feet by 60 feet.
Two of the sides of the pool are trapezoids.
The water is 8 feet deep at the deep end and 3 feet deep at the shallow end.


1. Find the volume of water in the pool. $\qquad$ cubic feet

Show your calculations.

The volume of water in the pool is 74,250 gallons.
2. A pump fills the pool at a rate of 1 gallon per second. How long, in hours and minutes, does it take to deliver 74,250 gallons of water into the pool?
$\qquad$ hours $\qquad$ minutes
Show your calculations.
3. (a) Which of these graphs best represents the depth of the water in the pool as it is filled at a steady rate of one gallon per second?

Graph $\qquad$

(b) Explain your reasons.
$\qquad$
$\qquad$
$\qquad$
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.

| Swimming Pool | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - work with trapezoids, rates and time graphs in a real context <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answer: $\mathbf{9 , 9 0 0}$ cubic feet <br> Shows correct calculation such as: $60 \times \underline{(8+3)} \times 30$ | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | 2 |
| 2. Gives correct answer: $\mathbf{2 0}$ hours, $\mathbf{3 7 . 5}$ minutes <br> Shows correct calculation: dividing 74,250 by $60 \times 60$ $=20.625$ hours | $\begin{aligned} & 1 \\ & 1 \\ & 1 \end{aligned}$ | 3 |
| 3. (a) Gives correct answer: Graph B <br> (b) Gives correct explanation such as: <br> At first the depth increases quickly, but then more slowly as the water moves up the slope. For the final 3 feet, the depth increases at a constant rate. <br> Partial credit <br> A partially correct explanation. | 1 <br> 2 <br> (1) | 3 |
| Total Points |  | 8 |

## Algebra

Course One/Algebra
Task 2
Odd Sums

| Student Task | Work with odd, even and consecutive numbers. Make and justify <br> conjectures about consecutive numbers. |
| :--- | :--- |
| Core Idea 2 <br> Mathematical <br> Reasoning | Employ forms of mathematical reasoning and proof appropriate to <br> the solution of the problem, including deductive and inductive <br> reasoning, making and testing conjectures and using <br> counterexamples and indirect proof. <br> $\bullet \quad$ Show mathematical reasoning in a variety of ways including |
|  | words, numbers, symbols, pictures, charts, graphs, tables, <br> diagrams, and models. |
|  | - Draw reasonable conclusions about a situation being modeled. |

Based on teacher observation, this is what algebra students knew and were able to do:

- Give examples to fit constraints using consecutive numbers
- Understand definitions of odd numbers, even numbers, and consecutive numbers
- Knew rules like an odd number plus and even number equals an odd number

Areas of difficulty for algebra students:

- Using algebra to make justifications
- Giving a process for finding the consecutive numbers rather than making a justification
- Noting the pattern of multiples of three in part four, rather than the more specific pattern of multiples of six
- Giving an explanation about the three consecutive numbers in part 4, instead of giving a rule for how to tell if the answer could be written as the sum of 3 consecutive numbers
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@ noycefdn.org.


## Odd Sums

This problem gives you the chance to:

- work with odd, even and consecutive numbers
- make and explain justifications

The odd number 9 can be written as the sum of two consecutive whole numbers.

$$
9=4+5
$$

The odd number 47 can be written as the sum of two consecutive whole numbers.

$$
47=23+24
$$

1. Show that the odd numbers 15 and 99 can be written as the sum of two consecutive whole numbers.

2. Explain why EVERY odd number can be written as the sum of two consecutive whole numbers.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@ noycefdn.org.

Even numbers cannot be written as the sum of two consecutive whole numbers.
Some even numbers can be written as the sum of three consecutive numbers.
For example: $\mathbf{2 4}=\mathbf{7 + 8 + 9}$
3. Find three other examples of even numbers that can be written as the sum of THREE consecutive whole numbers.
$\qquad$
$\qquad$
$\qquad$
4. Explain how you can tell whether an even number can be written as the sum of three consecutive whole numbers.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.

| Odd Sums | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - work with odd, even and consecutive numbers <br> - make and explain justifications <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answers: $\mathbf{7 + 8}$ $49+50$ | $1$ $1$ | 2 |
| 2. Gives a correct explanation, perhaps involving a description of how the sums can be made ('subtract 1 from the number, divided the result by 2 , and that gives the smaller of the two consecutive numbers in the sum') <br> Algebraically <br> All odd numbers can be written in the form $2 \mathrm{n}+1$, where n is an integer. And $2 \mathrm{n}+1=\mathrm{n}+(\mathrm{n}+1)$ | 2 | 2 |
| 3. Gives three correct examples. <br> Partial credit <br> Gives two correct examples. | 2 <br> (1) | 2 |
| 4. Gives a correct explanation such as: <br> (An even number that is) a multiple of 3 can be written as the sum of three consecutive whole numbers. <br> or <br> A multiple of 6 can be written as the sum of three consecutive whole numbers. | $\begin{gathered} 2 \\ \text { or } \\ 2 \end{gathered}$ | 2 |
| Total Points |  | 8 |

## Algebra

## Course One/Algebra

Task $3 \quad$ Patchwork Quilt

| Student Task | Recognize and extend a number pattern for a geometric pattern. Express a rule using algebra. Use inverse operations to solve a problem. |
| :---: | :---: |
| Core Idea 1 Functions and Relations | Understand patterns, relations, and functions. <br> - Generalize patterns using explicitly defined functions. <br> - Understand relations and functions and select, convert flexibly among, and use various representations for them. <br> - Recognize and generate equivalent forms of simple algebraic expressions and solve linear equations. |

Based on teacher observation, this is what algebra students knew and were able to do:

- Extend the pattern to five and explain how they got their answer, usually noting the growth rate of 5 .
- Work backwards to find the number of black hexagons needed for 66 white hexagons.

Areas of difficulty for algebra students:

- Writing a rule or formula
- Understanding the difference between a recursive rule and a generalized rule
- Understanding that a variable is not the same as a label
- Understanding how the first term is different or how the constant effects the progression of the pattern
- Expressing ideas in symbolic notation
- Order of operations


## Patchwork Quilt

This problem gives you the chance to:

- recognize and extend a number pattern
- express a rule using algebra

Sam is making a border for a patchwork quilt.
She is sewing black and white regular hexagons together.


Sam makes a table to show the number of black and white hexagons she needs.


1. How many white hexagons does Sam need for 6 black hexagons? $\qquad$
Explain how you figured it out.
$\qquad$
$\qquad$
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.
2. How many black hexagons does Sam need for 66 white hexagons?

Explain how you figured it out.
$\qquad$
$\qquad$
3. Write a formula that will help you to find how many white hexagons (W) Sam needs for $n$ black hexagons.
$\qquad$
4. Use your formula to find how many white hexagons Sam needs for 77 black hexagons.
$\qquad$
Show your work.

(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.

| Patchwork Quilt | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - recognize and extend a number pattern <br> - express a rule using algebra <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answer: $\mathbf{3 1}$ <br> Gives a correct explanation such as: <br> The first black hexagon needs 6 white hexagons and the other five black hexagons each need five white hexagons. $6+5 \times 5=31$ (accept $21+10=31$ ) | $1$ | 2 |
| 2. Gives correct answer: $\mathbf{1 3}$ <br> Gives a correct explanation such as: <br> The first black hexagon needs 6 white hexagons and the other black hexagons each need five white hexagons. $\begin{aligned} & 66-6=60 \\ & 60 \div 5=12 \\ & 12+1=13 \end{aligned}$ | 1 <br> 1 | 2 |
| 3. Gives correct answer: $\mathbf{W}=\mathbf{5 n} \mathbf{+ 1}$ or equivalent <br> Partial credit <br> Gives an expression such as $5 n+1$ | 2 <br> (1) | 2 |
| 4. Gives correct answer: $\mathbf{3 8 6}$ <br> Shows work such as: $\mathrm{W}=5 \times 77+1$ | $1$ $1$ | 2 |
| Total Points |  | 8 |

Algebra - 2006
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.

## Algebra

## Course One/Algebra

Printing Tickets

| Student Task | Compare price plans using graphs and formulae. Use inequalities in a <br> practical context of buying tickets. |
| :--- | :--- |
| Core Idea 3 | Represent and analyze mathematical situations and structures <br> Alg. Properties <br> using algebra. <br> $\bullet$ |
| Representations | Write equivalent forms of equations, inequalities and systems <br> of equations and solve them |
| -Use symbolic algebra to represent and explain mathematical <br> relationships |  |
| • Judge the meaning, utility, and reasonableness of results of |  |
| symbolic manipulations |  |

Based on teacher observation, this is what algebra students knew and were able to do:

- Write an equation for Best Print
- Draw a graph to match their equation
- Interpreting graphs of two equations to determine best buy under different conditions

Areas of difficulty for algebra students:

- Understand how to use symbolic notation to represent a context
- Find a table of values before drawing a graph
- Using algebra to solve for 2 equations with 2 unknowns
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.


## Printing Tickets

This problem gives you the chance to:

- compare price plans using graphs and formulae

Susie is organizing the printing of tickets for a show.
She has collected prices from several printers and these two seem to be the best.

## SURE PRINT

Ticket printing 25 tickets for $\$ 2$

| BEST PRINT |
| :---: |
| Tickets printed |
| $\$ 10$ setting up |
| plus |
| $\$ 1$ for 25 tickets |

1. Using $\mathbf{C}$ for the cost of the printing and $\mathbf{t}$ for the number of tickets, Susie writes a formula for each of the printers. Here is her formula for Sure Print:

$$
\text { Sure Print } \quad C=\frac{2 t}{25}
$$

Write the formula for Best Print:

$$
\text { Best Print } \quad \mathrm{C}=
$$

2. Susie's brother Rob has drawn Sure Print's graph on a grid.

Draw the graph for Best Print.

(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@ @oycefdn.org.
3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

$$
\mathrm{C}=\underline{\mathrm{t}=}
$$

Show how Susie may have calculated C and t .
4. What do Rob's graphs and Susie's calculations tell us about the cost of the tickets?

Which company should Susie choose under what circumstances?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

| 9 |
| :---: |
| 9 |


| Printing Tickets | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - compare price plans using graphs and formulae <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct formula such as: $\mathrm{C}=\mathbf{1 0 + t / 2 5}$ | 1 | 1 |
| 2. Draws a correct straight line from: $(\mathbf{0}, \mathbf{1 0})$ $\text { to }(400,26)$ | 2 | 2 |
| 3. Gives correct answers: $\begin{aligned} & C=\mathbf{2 0} \\ & t=\mathbf{2 5 0} \end{aligned}$ <br> Shows correct work such as: $\begin{aligned} & 2 t \div 25=10+t \div 25 \\ & 2 t=250+t \\ & C=2 \times 250 \div 25 \end{aligned}$ | 1 1 <br> 1 <br> 1 | 4 |
| 4. Gives a correct explanation such as: <br> If Susie buys less than 250 tickets, Sure Print will be cheaper, and if she buys more than 250 tickets, Best Print will be cheaper. | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | 2 |
| Total Points |  | 9 |

(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.

## Algebra

## Course One/Algebra

Task 5 Graphs
$\left.\begin{array}{|l|l|}\hline \text { Student Task } & \text { Relate line graphs to their equations. } \\ \hline \begin{array}{l}\text { Core Idea 3 Alg. } \\ \text { Properties \& } \\ \text { Representations }\end{array} & \begin{array}{l}\text { Represent and analyze mathematical situations and structures } \\ \text { using algebraic symbols. } \\ \text { - }\end{array} \\ & \begin{array}{l}\text { Understand the meaning of equivalent forms of expressions, } \\ \text { equations, inequalities, or relations }\end{array} \\ \text { - Write equivalent forms of equations, inequalities, and systems } \\ \text { of equations and solve them }\end{array}\right]$

Based on teacher observation, this is what algebra students knew and were able to do:

- Students could identify the origin
- Students could identify the equation $x+y=9$
- Students could identify the equation $y=1 / 2 x$
- Students could give the coordinates for the intersection of $y=6$ and $x=6$

Areas of difficulty for algebra students:

- Confusing the order of $x$ an $y$ in a coordinate pair
- Recognizing the solutions for simultaneous equations
- Writing an equation for a line through a given point
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.


## Graphs

This problem gives you the chance to:

- relate given line graphs to their equations

Here is a graphical diagram:

(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@ noycefdn.org.

1. Choose the correct label for each feature of the diagram from this list.

Write its letter in the correct place in the diagram. (Not all the letters are needed.)

A The line $y=0$
B The line $x=0$
C The line $x=6$
D The line $y=6$
E The origin
F The line $y=\frac{1}{2} x$
G The line $x+y=9$
H The line $y=x+6$
I The line $y=x-6$
J The solution of the simultaneous equations $x+y=9$ and $y=\frac{1}{2} x$
K The solution of the simultaneous equations $x+y=9$ and $y=2 x$
2. Which point is on the line $y=6$ and on the line $x=6$ ?
3. Write the equation of any straight line that goes through the point $(3,6)$.
$\qquad$
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@ noycefdn.org.

## Graphs

The core elements of performance required by this task are:

- relate given line graphs to their equations

| Based on these, credit for specific aspects of performance should be assigned as follows | points | section <br> points |
| :--- | :---: | :---: |
| 1.Gives 7 correct labels (see below) <br> Partial credit <br> 6 correct labels <br> 5 correct labels <br> 4 or 3 correct labels <br> 2 or 1 correct label( s$)$ | 5 |  |
| 2. $\quad$ Gives correct answer: (6,6) | $(3)$ |  |
| 3. | Gives any correct line, for example, $\mathrm{y}=6, \mathrm{x}=3, \mathrm{y}=\mathrm{x}+3, \mathrm{y}=9-\mathrm{x}$, etc. |  |
|  | 1 | 1 |



Algebra - 2006
(c) Noyce Foundation 2006. To reproduce this document, permission must be granted by the Noyce Foundation: info@noycefdn.org.

## Algebra

| Student Task | Work with linear and quadratic functions, their graphs, and equations. |
| :---: | :---: |
| Core Idea 1 <br> Functions and Relations | Understand patterns, relations, and functions. <br> - Analyze functions of one variable by investigating local and global behavior including slopes as rates of change, intercepts, and zeros. |
| Core Idea 3 <br> Algebraic <br> Properties and Representations | Represent and analyze mathematical situations and structures using algebraic symbols. <br> - Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations. <br> - Write equivalent forms of equations, inequalities and systems of equations and solve them. <br> - Use symbolic algebra to represent and explain mathematical relationships. |

## Mathematics in this task:

- Distinguish between linear and quadratic equations and their graphical representations
- Ability to graph a linear equation
- Ability to locate points on a graph and interpret their meaning
- Use algebra to find the intersections of two equations

Based on teacher observation, this is what algebra students knew and were able to do:

- Find the coordinates where the graphs intersect
- Give a reason for connecting equations with their graphs
- Draw a graph of $y=3 x$

Areas of difficulties for algebra students:

- Finding values for x , where one graph or equation is less than another
- Using algebra to find the points of intersection for two equations
- Knowing that the equations should equal each other at the points of intersection
- Using factoring as a tool to solve a quadratic equation
- Understanding that you can't divide by 0

Strategies used by successful students:

- Making a table of values to help them graph
- Understanding $\mathrm{y}=\mathrm{mx}+\mathrm{b}$ and using it to help them graph
- Substitution


## Graphs

This problem gives you the chance to:

- work with linear and quadratic functions their graphs and equations

This diagram shows the graphs of $y=x^{2}$ and $y=2 x$.


1. Fill in the labels to show which graph is which. Explain how you decided.
$\qquad$
$\qquad$
$\qquad$
2. Use the diagram to help you complete this statement:
$2 x$ is greater than $x^{2}$ when $x$ is between $\qquad$ and $\qquad$
3. The graphs of $y=x^{2}$ and $y=2 x$ cross each other at two points.
a. Write down the coordinates of these two points.
b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
4. 

a. On the diagram, draw the graph of $y=3 x$.
b. What are the coordinates of the points where $y=x^{2}$ and $y=3 x$ meet?
$\qquad$
c. Where do you think that the graphs of $y=x^{2}$ and $y=n x$ meet? $\qquad$
d. Use algebra to prove your answer.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

| Task 1: Graphs | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - work with linear and quadratic functions their graphs and equations <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Graphs correctly labelled and convincing reason given | 1 | 1 |
| 2. Gives correct answer: between $\mathbf{0}$ and $\mathbf{2}$ | 1 | 1 |
| 3.a Gives correct answer: $(\mathbf{0}, \mathbf{0})$ and $(\mathbf{2}, \mathbf{4})$ <br> b Shows correct reasoning to justify the answers in 3.a, such as: When the graphs meet, $\begin{aligned} & \boldsymbol{x}^{2}=\mathbf{2 x} \\ & \therefore x^{2}-2 x=0 \\ & x(x-2)=0 \end{aligned}$ <br> So $x=0$ or $x=2$ <br> When $x=0, y=0$ and when $x=2, y=4$ <br> So the coordinates are $(0,0)$ and $(2,4)$ | 1 <br> 1 <br> 1 | 3 |
| 4.a Correct graph drawn <br> b Gives correct answer: $(\mathbf{0}, \mathbf{0})$ and $(\mathbf{3}, \mathbf{9})$ <br> c Gives correct answer: $(\mathbf{0}, \mathbf{0})$ and $\left(\boldsymbol{n}, \boldsymbol{n}^{\mathbf{2}}\right)$ <br> d Shows correct work such as: When the graphs meet, $\begin{aligned} & x^{2}=n x \\ \therefore & x^{2}-n x=0 \\ & x(x-n)=0 \end{aligned}$ <br> So $x=0$ or $x=n$ <br> When $x=0, y=0$ and when $x=n, y=n^{2}$ <br> So the coordinates are $(0,0)$ and $\left(n, n^{2}\right)$ | 1 <br> 1 <br> 1 <br> 1 | 4 |
| Total Points |  | 9 |

## Algebra

House Prices

| Student <br> Task | Work with graphs and formulas in a real context. |
| :--- | :--- |
| Core Idea 5 <br> Data Analysis | Select and use appropriate statistical methods to analyze data. <br> $\bullet \quad$Understand the relationship between two sets of data, display <br> such data in a scatterplot, and describe trends and shape of the <br> plot including correlations (positive, negative, and no) and <br> lines of best fit. <br> Make inferences based on the data and evaluate the validity of <br> conclusions drawn. <br> Core Idea 3 <br> Algebraic <br> Properties and <br> Representations <br> Represent and analyze mathematical situations and structures <br> using algebraic symbols. <br> Use symbolic expressions to represent relationships arising <br> from various contexts. <br> Approximate and interpret rates of change, from graphic and <br> numeric data. |

Mathematic in this task:

- Understanding information on a scatterplot, looking for trends such as correlation or no correlation
- Recognizing that a linear function passing through the origin is a proportion and finding a formula for a proportion
- Reading and interpreting points on a graph
- Graphing inequalities

Based on teacher observations, this is what algebra students knew and were able to do:

- Read and locate points on a scatterplot to meet constraints of the context
- Recognize when there is no pattern in a scatterplot
- Describe a trend in a scatterplot

Areas of difficulty for algebra students:

- Finding a formula for a line on a graph
- Graphing an inequality on a graph from a verbal description


## House Prices

This problem gives you the chance to:

- work with graphs and formulas in a real context

In March 2006, a newspaper article reported that houses in Maryland are so expensive that many people are unable to afford the monthly house payments.

This graph shows the average house price and the average monthly payment for all the different counties in Maryland.

House Prices and Payments

1.
a. What does the pattern of the data indicate about the connection between house prices and monthly payments?
$\qquad$
$\qquad$
b. Find the monthly payment for a house costing $\$ 450000$.
c. Find a formula connecting the average monthly payment with the average house price.

This graph shows the average monthly wage and the average monthly house payment for each county in Maryland.

2.
a. Describe the pattern of the data.
b. Draw a ring round the point representing the county where the average person will find it most difficult to afford the monthly house payment. Label this point with the letter A.
c. Draw a ring round the point representing the county where the average person will find it easiest to afford the monthly house payment. Label this point with the letter B.
d. Indicate clearly which part of the graph contains points representing counties where the average monthly house payment is more than the average monthly wage.
$\qquad$

| Task 2: House Prices | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - work with graphs and formulas in a real context <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1.a Gives correct explanation such as: <br> There is a positive correlation between the two variables. <br> b Gives correct answer in the range $\$ 3400$ and $\$ 3800$.. <br> c Gives correct answer such as: $\mathrm{y}=0.008 \mathrm{x}$ (approximately) or $\mathrm{y}=\mathrm{x} / 125$, where $\$ x$ is the house price and $\$ y$ is the monthly payment or equivalent Accept an intercept in the range 0 to 100 . | 1 <br> 1 | 3 |
| 2.a Gives correct explanation such as: No correlation or equivalent <br> b Point A correctly indicated: $(2500,4450)$ <br> c Point B correctly indicated: $(2360,800)$, or $(3770,1270)$ <br> d Clear indication of correct region, above the line $y=x$ |  | 4 |
| Total Points |  | 7 |

## Ash's Puzzle

Algebra
Task 3
Ash's Puzzle

| Student Task | Find numbers that obey given rules or constraints. Find rules for sets <br> of numbers. Use understanding of place value to solve problems in <br> context. |
| :--- | :--- |
| Core Idea 3 <br> Algebraic <br> Properties and <br> Representations | Represent and analyze mathematical situations and structures <br> using algebraic symbols. <br> Use symbolic expressions to represent relationships <br> arising from various contexts. <br> Compare and contrast the properties of numbers and <br> number systems including real numbers |
| Core Idea 2 <br> Mathematical <br> Reasoning | Employ forms of mathematical reasoning and proof appropriate <br> to the solution of the problem, including deductive and inductive <br> reasoning, making and testing conjectures and using <br> counterexamples and indirect proof. <br> - |
|  | Show mathematical reasoning in a variety of ways, including <br> words, numbers, symbols, pictures, charts, graphs tables, <br> diagram and models. |
| -Explain the logic inherent in a solution process. <br> Use induction to make conjectures and use deductive <br> reasoning to prove conclusions. |  |
| - Draw reasonable conclusions about a situation being modeled. |  |

## Mathematics in this task:

- Investigating a relationship in number calculations
- Identifying relevant information using place value and number theory to discover a pattern in the solution
- Generalizing from arithmetic to a pattern for all solutions

Based on teacher observations, this is what algebra students knew and were able to do:

- Find most solutions which will reverse a two-digit number by adding nine
- Give examples of three-digit numbers that will reverse the digits when 99 is added and show supporting evidence

Areas of difficulty for algebra students:

- Making an organized list or check for "all" solutions that meet a set of constraints
- Testing different cases of numbers, investigating enough options or choices before making a generalization
- Recognizing all the relevant information needed to make a convincing set of rules for all numbers
- Vocabulary for place value


## Ash's Puzzle

This problem gives you the chance to:

- find numbers that obey given rules
- find rules for sets of numbers

Ash has a book of number puzzles. This is one of the puzzles.


1. Solve this puzzle for Ash.

Show that your answer works.

Ash wonders if there are other answers to this puzzle.
2. Are there other correct answers to the puzzle?

If there are more correct answers list them all. If not explain how you know that there is only one correct answer.
$\qquad$
$\qquad$
$\qquad$

Ash decides to try to find a three-digit number such that its digits are reversed when 99 is added. He finds that there are a lot of numbers that work.
3. Write four three-digit numbers that Ash could have found.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
Show your work.

Ash thinks that there must be rules that would make it possible to find all of the three-digit numbers that are reversed when 99 is added to them.
4. Find these rules for Ash.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

| Task 3: Ash's Puzzle | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - find numbers which obey given rules <br> - find rules for sets of numbers <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives a correct answer: $\mathbf{1 2}, \mathbf{2 3}, \mathbf{3 4}, \mathbf{4 5}, \mathbf{5 6}, \mathbf{6 7}, \mathbf{7 8}$ or $\mathbf{8 9}$ and <br> Gives correct calculation for their answer: such as $12+\mathbf{9}=21$ | 1 | 1 |
| 2. Gives correct answer: yes <br> and <br> lists the other seven possible answers (ignore their answer to question 1 repeated) $12,23,34,45,56,67,78$ and 89 <br> Partial credit: <br> An extra 4, 5 or 6 correct answers with no incorrect ones. | $2$ <br> (1) | 2 |
| 3. Gives 4 correct answers: any 3 digit numbers with the last digit 1 greater than the first e.g. 152,798 etc. <br> Shows some correct work for their answers such as: $152+99=251$ | $1$ | 2 |
| 4. Gives correct rules such as: <br> The last digit is one more than the first. <br> The middle digit can be any number. |  | 2 |
| Total Points |  | 7 |


| Student Task | Form expressions and solve an equations to solve an age problem. |
| :--- | :--- |
| Core Idea 3 | Represent and analyze mathematical situations and structures <br> using algebraic symbols. |
| Algebraic |  |
| Properties and | Use symbolic expressions to represent relationships <br> Representations |
| aring from various contexts. <br> Judge the meaning, utility, and reasonableness of results <br> of symbolic manipulations. |  |

Mathematics in this task:

- Write algebraic expressions to represent the relationships between students' ages
- Understand how to use symbolic notation to express distributive property for a multiplicative relationship
- Solve an equation
- Reason about elapsed time to find when Jan's age is double Will's age
- Identify constraints and use them to set up an equation

Based on teacher observations this is what algebra students know and are able to do:

- Write an algebraic expression for an additive relationship
- Find the ages for the three children

Areas of difficulty for algebra students:

- Writing a multiplicative expression that involves distributive property
- Writing and using an equation in a practical setting
- Understanding elapsed time and expressing elapsed time numerically or algebraically


## How Old Are They?

This problem gives you the chance to:

- form expressions
- form and solve an equation to solve an age problem

Will is $w$ years old.
Ben is 3 years older.

1. Write an expression, in terms of $w$, for Ben's age.

Jan is twice as old as Ben.
2. Write an expression, in terms of $w$, for Jan's age.

If you add together the ages of Will, Ben and Jan the total comes to 41 years.
3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

| Will is | years old |
| :---: | :---: |
| Ben is $\quad$ years old |  |
| Jan is | years old |

Show your work.
4. In how many years will Jan be twice as old as Will?
years

## Explain how you figured it out.

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

| Task 4: How Old Are They? | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - form expressions <br> - form and solve an equation <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives a correct expression: $\mathbf{w + 3}$ | 1 | 1 |
| 2. Gives a correct expression: $\mathbf{2}(\mathbf{w}+\mathbf{3})$ | 1 | 1 |
| 3 Gives correct answers: Will is $\mathbf{8}$ years old <br> Ben is $\mathbf{1 1}$ and Jan is $\mathbf{2 2}$ years old <br> Shows correct work such as: <br> $\mathrm{w}+\mathrm{w}+3+2(\mathrm{w}+3)$ (allow follow through) $\begin{aligned} & 4 w+9=41 \\ & 4 w=32 \end{aligned}$ | 1 <br> 1 <br> 1 ft . | 3 |
| 4. Gives a correct answer: in $\mathbf{6}$ years time <br> Gives a correct explanation such as: <br> Will is 14 years younger than Jan so when Will is 14 Jan will be 28. $14-8=6 \text {. }$ <br> Accept guess and check with correct calculations. <br> Solves correct equation. | 1 <br> 1 | 2 |
| Total Point |  | 7 |


| Student Task | Find solutions to equations and inequalities. |
| :--- | :--- |
| Core Idea 3 | Represent and analyze mathematical situations and structures <br> using algebraic symbols. <br> Algebraic |
| Properties and |  |
| Representations | Write equivalent forms of equations, inequalities and systems <br> of equations and solve them. <br> • Understand the meaning of equivalent forms of expressions, <br> equations, inequalities, or relations. |

Mathematics in this task:

- Ability to understand what is meant by the solution to an equation or inequality.
- Ability to calculate solutions to inequalities and equalities.
- Ability to use exponents, negative numbers and square roots.
- Ability to think about classes of numbers and the difference between finite and infinite sets
- Ability to recognize properties of equations and inequalities with regards to the number of possible solutions
- Ability to understand variable in a variety of ways

Based on teacher observations, this is what algebra students knew and were able to do:

- Find two solutions for $1776 x+1066 \geq 365$
- Find both solutions for $x^{2}=121$
- Find $x^{2}>x^{3}$ and for $|x|>x$

Areas of difficulty for algebra students:

- Understanding variable
- Understanding infinity
- Finding solutions for inequalities
- Identifying equations that have a limited number of solutions
- Thinking about classes of equations
- Working with exponents
- Understanding that the x's represent the same number with an equation


## Two Solutions

This problem gives you the chance to:

- find solutions to equations and inequalities

1. For each of the following equalities and inequalities, find two values for $x$ that make the statement true.
a. $\quad x^{2}=121$
b. $x^{2}=x$
c. $x^{2}<x$
d. $(x-1)\left(5 x^{4}-7 x^{3}+x\right)=0$
e. $1776 x+1066 \geq 365$
f. $\quad x^{2}>x^{3}$
g. $\quad|x|>x$
2. Some of the equations and inequalities on the page opposite have exactly two solutions; others have more than two solutions.
a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
c. Write down two equations or inequalities that have an infinite number of solutions.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

| Task 5: Two Solutions | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - find solutions to equations <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answers: <br> a: $\pm 11$ <br> b: 0,1 <br> c: any values between $\mathbf{0}$ and $\mathbf{1}$ <br> d: $\mathbf{0 , 1}$ <br> e: any value $\geq \mathbf{- 0 . 3 9 4 7}$ <br> f: any value less than 1 except 0 <br> g : any negative value | $7 \times 1$ | 7 |
| 2. Gives correct answers with reasons such as: <br> a. $\quad \mathbf{x}^{2}=\mathbf{1 2 1}$ and $\mathbf{x}^{2}=\mathbf{x}$ <br> These are quadratic equations with two roots <br> b. $(x-1)\left(5 x^{4}-7 x^{3}+x\right)=0$ <br> c. Gives two of: $\mathbf{x}^{2}<\mathrm{x}, 1776 \mathrm{x}+1066 \geq 365, \mathrm{x}^{2}>\mathrm{x}^{3},\|x\|>\boldsymbol{x}$ | 1 <br> 1 <br> 1 | 3 |
| Total Points |  | 10 |

## Expressions

| Student Task | Work with algebraic expressions for areas and perimeters of <br> parallelograms and trapezoids. |
| :--- | :--- |
| Core Idea 3 | Represent and analyze mathematical situations and structures <br> using algebraic symbols. |
| Algebraic |  |
| Properties and |  |
| Representations | Use symbolic algebra to represent and explain mathematical <br> relationships. |

Mathematics of this task:

- Using variables to find area and perimeter of a parallelogram
- Recognizing equivalent expressions by factoring or using distributive property
- Using algebra to make and prove a generalization
- Explaining the steps of factoring or using distributive property to make two expressions equivalent

Based on teacher observations, this is what algebra students know and are able to do:

- Students were able to recognize expressions for finding perimeter of a parallelogram
- Many students could recognize equivalent expressions for perimeter

Areas of difficulty for algebra students:

- Finding the area of a parallelogram/ translating from words to variables (the formula for area was given in a verbal form)
- Thinking with variables instead of numbers
- Decomposing the trapezoid into two triangles
- Using factoring and/or distributive. 0 property to make equivalent expressions
- Using algebra to make a generalization


## Expressions

This problem gives you the chance to:

- work with algebraic expressions for areas and perimeters of parallelograms and trapezoids

1. Here is a parallelogram.


The area of a parallelogram is the product of its base times the perpendicular height.
a. Which of these are correct expressions for the area of this parallelogram?

Draw a circle around any that are correct.

$$
a b \quad \frac{1}{2} a b \quad a h \quad \frac{1}{2} a h \quad 2 a+2 b \quad 2(a+b) \quad a b h
$$

b. Which of these are correct expressions for the perimeter of the parallelogram?

Draw a circle around any that are correct.
$a b$
$\frac{1}{2} a b$
$a h$
$\frac{1}{2} a h$
$2 a+2 b$
$2(a+b)$
$a b h$
2. Here is a trapezoid. It is made up of two triangles, each with height $h$.


Find the area of each of the two triangles and use your results to show that the area of the trapezoid is $\frac{1}{2}(a+b) h$.

| Expressions | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - work with algebraic expressions for areas and perimeters of parallelograms and trapezoids <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1.a Gives correct answer: $\boldsymbol{a} \boldsymbol{h}$ circled and no others circled <br> b. Gives correct answers: $2 a+2 b$ and $2(a+b)$ <br> Deduct 1 point for 1 extra and 2 points for more than 1 extra. | $\begin{gathered} 1 \\ 2 \times 1 \end{gathered}$ | 3 |
| 2. Provides a convincing development of the required expression such as: Shows the areas of the two triangles are $\frac{1}{2} a h$ and $\frac{1}{2} b h$ Adds these two expressions to get $\frac{1}{2}(a+b) h$ | $2 \times 1$ | 3 |
| Total Points |  | 6 |


| Student Task | Form and solve a pair of linear equations in a practical situation. |
| :--- | :--- |
| Core Idea 3 | Represent and analyze mathematical situations and structures <br> Algebraic |
| using algebraic symbols. |  |
| Properties and |  |
| Representations | $\quad$Use symbolic algebra to represent and explain mathematical <br> relationships. |
|  | Judge the meaning, utility, and reasonableness of results of <br> symbolic manipulation. |

Mathematics of the task:

- Understanding equalities and maintaining equalities to set up and solve equations
- Defining variables
- Combining like terms, not combining unlike terms
- Understanding monetary units
- Understanding that two linear equations with two unknowns has a unique solution, but that a linear function with two variables has an infinite number of solutions
- Strategies or procedures for solving sets of equations: substitution, multiplication/addition

Based on teacher observations, this is what algebra students know and are able to do:

- Write an equation from context and understand what the variables represent
- Use guess and check successfully as a strategy to find a solution for two equations with two unknowns.
- Use substitution or multiplication/ addition to solve for the unknowns

Areas of difficulty for algebra students:

- Quantifying answers to justify a conjecture
- Using distributive property correctly in solving a problem
- Identifying which equation and which unknown would be easiest to use when applying the substitution method
- Using partially remembered strategies, but could not carry them through the entire solution process or unsuccessfully combined strategies
- Checking work with both equations to see if the solution is true for both
- Understanding that functions have multiple solutions


## Buying Chips and Candy

This problem gives you the chance to:

- form and solve a pair of linear equations in a practical situation

Ralph and Jody go to the shop to buy potato chips and candy bars.


Ralph buys 3 bags of potato chips and 4 candy bars. He spends $\$ 3.75$.
Jody buys 4 bags of potato chips and 2 candy bars. She spends $\$ 3.00$.
Later Clancy joins Ralph and Jody and asks to buy one bag of potato chips and one candy bar from them. They need to work out how much he should pay.

Ralph writes

$$
3 p+4 b=375
$$

1. If $\boldsymbol{p}$ stands for the cost, in cents, of a bag of potato chips and $\boldsymbol{b}$ stands for the cost, in cents, of a candy bar, what does the 375 in Ralph's equation mean?
$\qquad$
$\qquad$
$\qquad$
2. Write a similar equation, using $\boldsymbol{p}$ and $\boldsymbol{b}$, for the items Jody bought.
3. Use the two equations to figure out the price of a bag of potato chips and the price of a candy bar.

Potato chips $\qquad$
Candy bar $\qquad$ Show your work.
4. Clancy has just $\$ 1$. Does he have enough money to buy a bag of potato chips and a candy bar?

Explain your answer by showing your calculation.
$\qquad$
$\qquad$
$\qquad$

| Buying Chips and Candy | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - form and solve a pair of linear equations in a practical situation <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives a correct explanation such as: <br> It stands for the 375 cents that Ralph spent. (Must have correct units) | 1 | 1 |
| 2. Writes a correct equation such as: $\mathbf{4 p}+\mathbf{2 b}=\mathbf{3 0 0}$ <br> Partial credit <br> For an almost correct equation. ( Left hand side of equation must be correct) | $2$ <br> (1) | 2 |
| 3. Gives correct answers: $\mathbf{4 5}$ cents or $\mathbf{\$ 0 . 4 5}$ and $\mathbf{6 0}$ cents or $\$ \mathbf{0 . 6 0}$ <br> Shows correct work such as: $\begin{aligned} & 8 p+4 b=6 \\ & \text { subtract } 3 p+4 b=375 \\ & 5 p=225 \\ & P=45 \\ & 4 \times 45+2 b=300 \\ & 2 b=120 \\ & b=60 \end{aligned}$ <br> Partial credit <br> For some correct work. | 1 ft <br> 2 ft <br> (1 ft) | 3 |
| 4. Gives a correct answer: no and <br> Shows a correct calculation such as: $0.60+0.45=1.05$ | 1 ft | 1 |
| Total Points |  | 7 |

Algebra Task 3 Sorting Functions

| Student Task | Find relationships between graphs, equations, tables and rules. <br> Explain your reasons. |
| :--- | :--- |
| Core Idea 1 | Understand patterns, relations, and functions. <br> Functions <br> and Relations |
| Understand relations and functions and select, convert flexibly <br> among, and use various representations for them. |  |
| Algebraic <br> Properties and <br> Representations | Represent and analyze mathematical situations and structures <br> using algebraic symbols. <br> • Use symbolic algebra to represent and explain mathematical <br> relationships. |
| • Judge the meaning, utility, and reasonableness of results of |  |
| symbolic manipulation. |  |

The mathematics of this task:

- Making connections between different algebraic representations: graphs, equations, verbal rules, and tables
- Understanding how the equation determines the shape of the graph
- Developing a convincing argument using a variety of algebraic concepts
- Being able to move from specific solutions to thinking about generalizations

Based on teacher observations, this is what algebra students know and are able to do:

- Understand that squaring a variable yields a parabola and that the variable that is squared effects the axis around which the parabola divided
- Use process of elimination as a strategy
- Match equations to tables and graphs
- Look for intercepts as a strategy
- Use vocabulary, such as: parabola, intercept, and linear

Areas of difficulty for algebra students:

- Knowing the difference between linear and non-linear equations
- Not knowing how to explain how they matched the graph and the equation
- Connecting the constant to the slope, e.g. just because it's -2 doesn't meant it's negative slope
- Quantifying: even though they could describe the process, but didn't quantify
- Not knowing how or when to use the term "curve" or parabola


## Sorting Functions

This problem gives you the chance to:

- Find relationships between graphs, equations, tables and rules
- Explain your reasons

On the next page are four graphs, four equations, four tables, and four rules.
Your task is to match each graph with an equation, a table and a rule.

1. Write your answers in the following table.

| Graph | Equation | Table | Rule |
| :---: | :--- | :--- | :--- |
| A |  |  |  |
| B |  |  |  |
| C |  |  |  |
| D |  |  |  |

2. Explain how you matched each of the four graphs to its equation.

Graph A $\qquad$
$\qquad$
$\qquad$
Graph B $\qquad$
$\qquad$
$\qquad$
Graph C $\qquad$
$\qquad$
$\qquad$
Graph D $\qquad$
$\qquad$
$\qquad$


## Sorting Functions



## Algebra

## Sidewalk Patterns

| Student Task | Work with patterns. Work out the nth term of a sequence. |
| :---: | :---: |
| Core Idea 1 Functions and Relations | Understand patterns, relations, and functions. <br> - Understand relations and functions and select, convert flexibly among, and use various representations for them. |
| Core Idea 3 <br> Algebraic <br> Properties and <br> Representations | Represent and analyze mathematical situations and structures using algebraic symbols. <br> - Use symbolic algebra to represent and explain mathematical relationships. <br> - Judge the meaning, utility, and reasonableness of results of symbolic manipulation. |

Mathematics of the task:

- Drawing and extending a visual pattern
- Using a table to extend a pattern
- Noticing relationships and number patterns, such as perfect squares
- Writing an expression to give the nth term in a sequence
- Working backwards, being able to "do and undo" a computational procedure

Based on teacher observations, this is what algebra students know and are able to do:

- Recognize squares, square numbers
- Draw and extend a visual pattern
- Extend a pattern by completing a table
- Compare the relationship between white and gray blocks by making observations from the table
- Find square roots of numbers

Areas of difficulty for algebra students:

- Writing an equation or making a generalized rule
- Considering all the relationships in the pattern
- How to algebraically express an even number
- How to define the "added" number
- How to explain the difference in the blocks given the total, what to do with the $1 / 2$


## Sidewalk Patterns

This problem gives you the chance to:

- work with patterns
- work out the $\mathrm{n}^{\text {th }}$ term of a sequence

In Prague some sidewalks are made of small square blocks of stone.
The blocks are in different shades to make patterns that are in various sizes.


Pattern number 1


Pattern number 2


Pattern number 3

1. Draw the next pattern in this series.


You may not need to use all of the squares on this grid.

Pattern number 4
Algebra - 2008
Copyright © 2008 by Noyce Foundation
All rights reserved.
2. Complete the table below.

| Pattern number, n | 1 | 2 | 3 | 4 |
| :--- | :---: | :---: | :---: | :---: |
| Number of white blocks | 12 | 40 |  |  |
| Number of gray blocks | 13 |  |  |  |
| Total number of blocks | 25 |  |  |  |

3. What do you notice about the number of white blocks and the number of gray blocks?
4. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.
a. Fill in the blank spaces in this list.

$$
25=5^{2} \quad 81=\ldots \quad 169=\ldots 17^{2}
$$

b. How many blocks will pattern number 5 need?
c. How many blocks will pattern number $n$ need?
5. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.
$\qquad$
$\qquad$
b. Pattern number 6 has a total of 625 blocks. How many white blocks are needed for pattern number 6? Show how you figured this out.


## Functions

| Student Task | Work with graphs and equations of linear and non-linear functions. |
| :---: | :---: |
| Core Idea 1 Functions and Relations | Understand patterns, relations, and functions. <br> - Understand relations and functions and select, convert flexibly among, and use various representations for them. |
| Core Idea 3 <br> Algebraic <br> Properties and <br> Representations | Represent and analyze mathematical situations and structures using algebraic symbols. <br> - Use symbolic algebra to represent and explain mathematical relationships. <br> - Judge the meaning, utility, and reasonableness of results of symbolic manipulation. |

The mathematics of this task:

- Identify linear points on a coordinate grid and name them
- Write an equation for a linear function from a graph or from coordinates
- Recognize non-linear points that form a parabola and estimate the graph of the curve
- Distinguish between features of a linear, quadratic and exponential graph and their equations
- Find the equation for a parabola given some of the coordinate points

Based on teacher observations, this is what algebra students knew and were able to do:

- Understand that a linear graph is a straight line
- Know that a non-linear graph is a parabola
- Identifying points on a graph

Areas of difficulty for algebra students:

- Finding a linear equation from a graph
- Finding a quadratic equation
- Drawing a parabola
- Difficulty in knowing difference between quadratic and quadrilateral (four points/four sides)
- Confusion about quadratic and exponential equations ( $\mathrm{x}^{2}$ has an exponent so it was exponential)
Strategies used by successful students:
- Knowing the generic formula for a line: $\mathrm{y}=\mathrm{mx}+\mathrm{b}$ and using slope and substitution
- Looking for the y-intercept for the linear equation
- Finding slope
- Knowing the generic formula for a quadratic: $y=a x^{2}+b x+c$
- Knowing mathematical vocabulary: quadratic, exponential


## Functions

This problem gives you the chance to:

- work with graphs and equations of linear and non-linear functions

On the grid are eight points from two different functions.

- four points fit a linear function
- the other four points fit a non-linear function.

For the linear function:
1.Write the coordinate pairs of its four points.
$\qquad$
$\qquad$
$\qquad$
$\qquad$


Draw the line on the grid.
2. Write an equation for the function.

Show your work.
$\qquad$

For the non-linear function:
3. Write the coordinate pairs of its four points.
$\qquad$
$\qquad$
$\qquad$

Algebra - 2008

Draw the graph of the function on the grid.
4.


Who is correct?
Explain your reasons.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
5. Write an equation that fits the non-linear function.

Show your work.

| Functions | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - work with graphs and equations of linear and non-linear functions <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answers: $(\mathbf{2}, \mathbf{9}),(\mathbf{3}, \mathbf{7}),(\mathbf{4}, \mathbf{5}),(\mathbf{5}, \mathbf{3})$ and Draws a correct line on the grid. | 1 | 1 |
| 2. Gives correct answer: $\mathbf{y}=\mathbf{1 3 - 2 x}$ | 2 | 2 |
| 3. Gives correct answers: $(\mathbf{1}, \mathbf{5}),(\mathbf{2}, \mathbf{8}),(\mathbf{3}, \mathbf{9}),(\mathbf{4}, \mathbf{8})$ <br> Draws a correct curved graph or equivalent | $\begin{aligned} & 1 \\ & 1 \end{aligned}$ | 2 |
| 4. Gives correct answer: Chris <br> and <br> Gives a correct explanation such as: <br> The graph has a turning point. or It is part of a parabola. | 1 | 1 |
| 5. Gives correct answer: $y=6 x-\mathbf{x}^{2}$ or equivalent such as $-(x-3)^{2}+9$ <br> Shows some correct work such as: <br> Substitutes coordinates in $\mathrm{y}=\mathrm{ax}^{2}+\mathrm{bx}+\mathrm{c}$ |  | 2 |
| Total Points |  | 8 |


| Student Task | Make an equation and solve a problem. |
| :--- | :--- |
| Core Idea 3 | Represent and analyze mathematical situations and structures <br> Algebraic <br> using algebraic symbols. |
| Properties and <br> Representations | Understand the meaning of equivalent forms of expressions, <br> equations, inequalities, or relations. |
|  | Write equivalent forms of equations, inequalities and systems <br> of equations and solve them. |
|  | Use symbolic algebra to represent and explain mathematical <br> relationships. |

## Mathematics of this task:

- Writing an expression from a diagram
- Understanding the relationship of equality in the context of a balanced scale
- Using equivalent ratios in context
- Distinguishing between a numerical coefficient and a value for the variable
- Understanding when there is sufficient information to solve for a variable
- Understanding meaning of variable or unknown

Based on teacher observations, this is what algebra students know and are able to do:

- Write expressions for diagrams
- Write equations for the diagram

Areas of difficulties for algebra students:

- Trying to assign values to x and y
- Trying to combine like terms from different sides of an equality
- Understanding the concept of equality illustrated in the diagram
- Using relational thinking to find equivalent ratios


## Soup and Beans

This problem gives you the chance to:

- make an equation and solve a problem

The weight of one can of beans is $x$ ounces.
The weight of one can of soup is y ounces.

1. Write an expression for the weight of the cans on the left hand side of the weighing scales.
2. Write an expression for the weight of the cans on the right hand side of the weighing scales.

3. Write an equation that shows the relationship between $x$ and $y$.
4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

Show your work.


## 2009 Rubrics



## Quadratic

| Student Task | Work with a quadratic function in various forms. |
| :--- | :--- |
| Core Idea 3 | Represent and analyze mathematical situations and structures <br> using algebraic symbols. <br> Algebraic |
| Properties and <br> Representations | Approximate and interpret rates of change, from graphic and <br> numeric data. |
| Core Idea 1 <br> Functions and <br> Relations$\quad$Understand relations and functions and select, convert flexibly <br> among, and use various representations for them. |  |

Mathematics of the task:

- Calculations with exponents and negative numbers
- Codifying calculations into a symbolic string
- Rectifying two forms of an equation
- Interpreting graphical representations of linear and quadratic equations and identifying minimum point and solutions
- Using algebra to find the solution to a quadratic equation (using factoring or the quadratic equation)

Based on teacher observation, this is what algebra students know and are able to do:

- Calculate using the number machine
- Identify the minimum point on a parabola using a graph

Areas of difficulty for algebra students:

- Using algebra to show that two equations are equal
- Finding solutions to two equations on a graph
- Using algebra to find the solutions to a quadratic equation


## Quadratic

This problem gives you the chance to:

- work with a quadratic function in various forms

This is a quadratic number machine.


1. a. Show that, if $x$ is $5, y$ is 7 .
b. What is $y$ if $x$ is 0 ?
c. Use algebra to show that, for this machine, $y=x^{2}-2 x-8$. $\qquad$
$\qquad$
$\qquad$

The diagram on the next page shows the graph of the machine's quadratic function $y=x^{2}-2 x-8$ and the graphs of $y=3$ and $y=x$.
2. a. Which point on the diagram shows the minimum value of $y$ ?
b. Which point(s) on the diagram show(s) the solution(s) to the equation $3=x^{2}-2 x-8$ ?
$\qquad$
c. Which point(s) on the diagram show(s) the solution(s) to the equation $x=x^{2}-2 x-8$ ?
$\qquad$
3. a. Use the graph to solve the equation $x^{2}-2 x-8=0$. Mark the solutions on the graph.

$$
\mathrm{x}=\square \quad \text { or } \quad \mathrm{x}=
$$

b. Use algebra to solve the same equation.
$\qquad$
$\qquad$
$\qquad$
$\qquad$


| Quadratic | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - work with a quadratic function in various forms <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. a. Gives a correct answer: $5 \rightarrow 4 \rightarrow 16 \rightarrow 7$ <br> b. Gives a correct answer: - $\mathbf{8}$ <br> c. Gives a correct answer: $\begin{aligned} y & =(x-1)^{2}-9 \\ & =x^{2}-2 x+1-9 \\ & =x^{2}-2 x-8 \end{aligned}$ | 1 <br> 1 <br> 2 | 4 |
| 2. a. Gives a correct answer: C <br> b. Gives a correct answer: A and E <br> c. Gives a correct answer: B and D | $\begin{aligned} & 1 \\ & 1 \\ & 1 \end{aligned}$ | 3 |
| 3. a. Gives a correct answer: -2, 4 <br> b. Gives a correct answer such as: $(x+2)(x-4)=0$, so $x=-2$ or 4 or uses completing the square to find 1 or 2 correct answers or uses quadratic formula. | 1 <br> 1 | 2 |
| Total Points |  | 9 |

## Circles and Spheres

| Student Task | $\begin{array}{l}\text { Find numbers that obey given rules or constraints. Find rules for sets } \\ \text { of numbers. Use understanding of place value to solve problems in } \\ \text { context. }\end{array}$ |
| :--- | :--- |
| $\begin{array}{l}\text { Core Idea 3 } \\ \text { Algebraic } \\ \text { Properties and } \\ \text { Representations }\end{array}$ | $\begin{array}{l}\text { Represent and analyze mathematical situations and structures } \\ \text { using algebraic symbols. } \\ \text { - Use symbolic expressions to represent relationships } \\ \text { arising from various contexts. } \\ \text { Compare and contrast the properties of numbers and } \\ \text { number systems including real numbers }\end{array}$ |
| $\begin{array}{l}\text { Core Idea 2 } \\ \text { Mathematical } \\ \text { Reasoning }\end{array}$ | $\begin{array}{l}\text { Employ forms of mathematical reasoning and proof appropriate } \\ \text { to the solution of the problem, including deductive and inductive } \\ \text { reasoning, making and testing conjectures and using } \\ \text { counterexamples and indirect proof. } \\ - \\ \text { Show mathematical reasoning in a variety of ways, including } \\ \text { words, numbers, symbols, pictures, charts, graphs tables, } \\ \text { diagram and models. }\end{array}$ |
| - Draw reasonable conclusions about a situation being modeled. |  |$\}$

Mathematics of the task:

- Know and be able to calculate volume, area, and circumference
- Reason about shapes of graphs, particularly relative to linear functions and quadratic or cubic functions
- Solve equations for different variables

Based on teacher observations, this is what algebra students knew and were able to do:

- Recognize that volume would show the steepest growth curve
- Match the graphs to the descriptions
- Solve area formula for the radius

Areas of difficulty for algebra students:

- Explaining why the descriptions matched the graphs
- Giving quantities for some measures, but not for all to make the comparison
- Solving a formula for a different variable


## Circles and Spheres

This problem gives you the chance to:

- compare functions for length, area and volume
- rearrange formulas
1.The diagram shows the graphs of three functions:
- Area of circle against radius
- Volume of sphere against radius
- Circumference of circle against radius

The formula to calculate the volume of a sphere from its radius is

$$
\mathrm{V}=4 / 3 \pi r^{3}
$$

where $r$ is the radius and
V is the Volume

In each case, the radius varies from 0 to 5 units,


For each letter, choose the correct description of the graph from the list above.
A
B $\qquad$
C $\qquad$
Say how you figured it out.
2. The formula to calculate the area of a circle from its radius is $A=\pi r^{2}$, where $r$ units is the radius and $A$ square units is the area.

Which of the formulas below works out the radius of a circle from its area?
Say 'yes' or 'no' for each one.

$$
\begin{aligned}
& r=\sqrt{\frac{\pi}{A}} \\
& r=\frac{A}{2 \pi} \\
& r=\frac{2 \pi}{A} \\
& r=\sqrt{\frac{A}{\pi}}
\end{aligned}
$$

3. The formula to calculate the circumference of a circle from its radius is $C=2 \pi r$, where $r$ units is the radius and C units is the circumference.

Find a formula to work out the radius of a circle from its circumference.
$\qquad$
$\qquad$
$\qquad$

## Circles and Spheres

Rubric
The core elements of performance required by this task are:

- compare functions for length, area and volume
- rearrange formulas

Based on these, credit for specific aspects of performance should be assigned as follows

1. Gives correct answers:

## A: Volume of sphere against radius

B: Area of circle against radius
section
points

C: Circumference of circle against radius
Gives correct explanations such as:
C shows a linear relationship so represents length
$B$ and A must represent the quadratic relationship for area and the cubic relationship for volume - comparing them shows $B$ is the quadratic and $C$ is the cubic graph.

May calculate some values such as:

| radius | volume | area | circumference |
| :---: | :---: | :---: | :---: |
| 3 | 113 | 28 | 19 |
| 4 | 268 | 50 | 25 |
| 5 | 523 | 79 | 31 |

## Partial credit

1 error

|  |  |  | 5 |
| :---: | :---: | :---: | :---: |
| 2. Gives correct answers: No, no, no, yes |  | 2 | 2 |
| Partial credit |  |  |  |
| One error |  | (1) |  |
| 3. Gives correct answer: $r=\frac{C}{2 \pi}$ |  | 1 |  |
|  |  |  | 1 |
|  | Total Points |  | 8 |


| Student Task | Form equations that match situations expressed in words. |
| :--- | :--- |
| Core Idea 3 | Represent and analyze mathematical situations and structures <br> using algebraic symbols. |
| Algebraic <br> Properties and <br> Representations | Use symbolic expressions to represent relationships <br> arising from various contexts. |
|  | Judge the meaning, utility, and reasonableness of results <br> of symbolic manipulations. |
|  | Understand the meaning of equivalent forms of <br> expressions, equations, inequalities or relations. |

## Mathematics of this task:

- Use symbolic expressions to describe relationships from verbal descriptions
- Use inverse operations in context
- Write equations and navigate between different representations
- Understand and choose appropriate operations to match a context

Based on teacher observations, this is what algebra students know and are able to do:

- Work with operations of addition and subtraction in context
- Manipulate equations to find equivalents

Areas of difficulty for algebra students:

- Forgetting negative sign when moving variable to a different side of an equation (working with inverse operations)
- Understanding order in a subtractions context (What is the whole? What is the part?)
- Choosing multiplication and division in context
- Understanding order in a division context (What is the quantity being divided? What is it being divided by?)


## Words and Equations

This problem gives you the chance to:

- write equations that match situations expressed in words

On this page are five situations expressed in words.
On the next page are ten algebraic equations.
Your task is to match each word situation with two algebraic equations.

| Word situation | Matches two equations |
| :--- | :--- | :--- |
| Cousins <br> Maria is 12 years older than her cousin Andy. <br> Maria is $x$ years old. <br> Andy is $y$ years old. | Equation 1 |
| Walking <br> Tom is walking 12 miles from A to B <br> He has already walked $x$ miles. <br> He needs to walk y miles more. | Equation 2 |
| Boxes and bottles | Equation 1 |
| There are twelve bottles in each box. | Equation 1 |
| The total number of bottles is x. | Equation 2 |
| The number of boxes is y. | Equation 1 |
| Temperatures |  |
| It is $12^{\circ}$ colder in the mountain than in the valley. |  |
| The temperature on the mountain is $x^{\circ}$ |  |
| The temperature in the valley is $y^{\circ}$. | Equation 2 |
| Years and months |  |
| There are twelve months in every year. |  |
| The number of years is x. |  |
| The number of months is y. |  |


| $x=12 y$ | $y=x+12$ |
| :---: | :---: |
| $y=x-12$ | $x=y+12$ |
| $x=y-12$ | $y=12 x$ |
| $y=12-x$ | $y=\frac{x}{12}$ |
| $x=\frac{y}{12}$ | $x=12-y$ |
|  |  |


| Words and Equations | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - write equations that match situations expressed in words <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answer: $\mathbf{x}=\mathbf{y}+\mathbf{1 2}$ and $\mathbf{y}=\mathbf{x} \mathbf{- 1 2}$ Accept: $x-y=12$ or equivalent answers. | 2 $\times 1$ | 2 |
| 2. Gives correct answer: $\mathbf{x}=\mathbf{1 2 - \mathbf { y }}$ and $\mathbf{y}=\mathbf{1 2} \mathbf{- x}$ Accept: $x+y=12$ or equivalent answers. | 2 x 1 | 2 |
| 3. Gives correct answer: $\mathbf{x}=\mathbf{1 2} \mathbf{y}$ and $\mathbf{y}=\mathbf{x} / \mathbf{1 2}$ Accept: $x / y=12$ or equivalent answers. | 2 x 1 | 2 |
| 4. Gives correct answer: $\mathbf{x}=\mathbf{y}-\mathbf{1 2}$ and $\mathbf{y}=\mathbf{x + 1 2}$ Accept: $y$ - $x=12$ or equivalent answers. | $2 \times 1$ | 2 |
| 5. Gives correct answer: $\mathbf{x}=\mathbf{y} / \mathbf{1 2}$ and $\mathbf{y}=\mathbf{1 2 x}$ Accept: $y / x=12$ or equivalent answers. | $2 \times 1$ | 2 |
| Total Points |  | 10 |


| Student Task | Use a chart to solve simultaneous equations. |
| :--- | :--- |
| Core Idea 3 | Represent and analyze mathematical situations and structures <br> using algebraic symbols. <br> Algebraic |
| Properties and |  |
| Representations | Write equivalent forms of equations, inequalities and systems <br> of equations and solve them. |
|  | Understand the meaning of equivalent forms of expressions, <br> equations, inequalities, or relations. |

## Mathematics of this task:

- Read and interpret a diagram
- Write equations from a graphical representation
- Solve 2 equations with 2 unknowns to find a unit price
- Use proportional reasoning or equations to find the cost of an amount not on the table

Based on teacher observations, this is what algebra students knew and were able to do:

- Read and interpret the table to describe the 500 using appropriate units
- Find the cost in part 2
- Use guess and check to find the unit prices in part 2

Areas of difficulty for algebra students:

- Using the information in the table to write equations
- Solving simultaneous equations or recognizing the using equations could be a strategy to find the unit prices
- Trying to impose a unit of scale on the table without considering all the information


## Coffee

This problem gives you the chance to:

- use a chart to solve simultaneous equations

This chart shows the cost, in cents. of different numbers of small and large cups of coffee.


1. Explain what the number 500 in the chart means.
2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

Small cup of coffee costs $\qquad$ cents

Large cup of coffee costs $\qquad$ cents

[^1]3. What number should go in the empty box that the arrow is pointing to. Explain your work.

| Coffee | Rubric |  |
| :---: | :---: | :---: |
| The core elements of performance required by this task are: <br> - use a chart to solve simultaneous equations <br> Based on these, credit for specific aspects of performance should be assigned as follows | points | section points |
| 1. Gives correct answer such as: the cost of 3 small cups of coffee and 2 large cups of coffee is $500 ¢$. | 1 | 1 |
| 2. Gives correct answer: small $\mathbf{8 0}$ ¢, large $\mathbf{1 3 0}$ ¢ <br> Shows work such as: solving simultaneous equations <br> Partial credit <br> Some correct work | $\begin{gathered} 2 \times 1 \\ 2 \end{gathered}$ | 4 |
| 3. Gives correct answer: $\mathbf{3 4 0}$ <br> and <br> Gives correct explanation such as: <br> The cost of one small cup and two large cups. (is half the cost of two small cups and four large cups.) | 2 ft | 2 |
| Total Points |  | 7 |


[^0]:    Course One - 2005
    pg. $\quad 40$

[^1]:    Algebra
    Coffee
    Copyright © 2009 by Mathematics Assessment Resource Services. All right reserved.

