#### MARS Tasks | Course 1

Page	Name of MARS Task	Year	Math Strand	Notes
*	Vacuum Cleaning	2003	GM	Draw, identify regions in context
*	Snakes	2003	DA	Interpret scatter plots, identify values
*	Crisscross Numbers	2003	PFA	Use algebra to explain number patterns
*	Conference Tables	2003	PFA, GM	Find/extend pattern in geometric context
*	Number Towers	2003	PFA	Form/solve equations from number pattern
*	Square Patterns	2004	PFA	Work with patterns, write formula
*	Population	2004	DA, PFA	Interpret scatter plot, perform operations
*	From 2 to 3 Dimensions	2004	GM	Imagine 3-D from 2-D net, compare
*	Graphs	2004	PFA	Relate equations, descriptions, graphs
*	Fibonacci Sequences	2004	PFA	Use algebra to solve number problems
2	Magic Squares	2005	PFA	Calculate values using algebraic notation
5	Vacations	2005	PFA	Analyze relationships w/graphs, algebra
9	Multiples of Three	2005	PFA	Test statement, find examples, justify
13	Scatter Diagram	2005	DA	Discuss, understand scatter plot
17	Fraction Sequences	2005	PFA	Extend sequence with fractions, decimals
21	Swimming Pool	2006	GM, PFA	Work w/trapezoids, rates, and time graphs
25	Odd Sums	2006	PFA	Word w/odd, even & consecutive numbers
29	Patchwork Quilt	2006	GM, PFA	Extend pattern to express rule using algebra
33	Printing Tickets	2006	PFA	Compare prices using graphs, formulas
37	Graphs	2006	PFA	Relate line graphs to equations
41	Graphs	2007	PFA	Work with functions, graphs, equations
45	House Prices	2007	PFA	Graphs, formulas in real context
49	Ash's Puzzle	2007	PFA	Find numbers that obey rules, find rules
53	How Old Are They?	2007	PFA	Form/solve equation to solve age problem
57	Two Solutions	2007	PFA	Find solutions to equations, inequalities
61	Expressions	2008	PFA	Algebraic expressions for shapes
64	Buying Chips and Candy	2008	PFA	Form/solve linear equations in context
68	Sorting Functions	2008	PFA	Find/explain graphs, equations, tables, rules
72	Sidewalk Patterns	2008	PFA	Work with patterns, sequences
76	Functions	2008	PFA	Work with linear/non-linear functions
80	Soup and Beans	2009	PFA	Make equation to solve problem
83	Quadratic	2009	PFA	Work with quadratic function
87	Circles and Spheres	2009	PFA	Compare functions: length, area, volume
91	Words and Equations	2009	PFA	Write equations to match situations
99	Coffee	2009	PFA	Use chart to solve simultaneous equations

NP=Number Properties NO=Number Operations PFA=Patterns Functions Algebra GM=Geometry & Measurement DA=Data Analysis

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Course 1	Task 1Magic Squares
Student Task Core Idea 3 Algebraic Properties and Representations	<ul> <li>Use symbolic algebraic notation to calculate values in "magic" squares where each row, column and diagonal adds to the same number.</li> <li>Represent and analyze mathematical situations and structures using algebraic symbols.</li> <li>Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations</li> <li>Write equivalent forms of equations, inequalities and systems of equations and solve them</li> <li>Use symbolic algebra to represent and explain methometical model and explain</li> </ul>
Core Idea 2 Mathematical Reasoning	<ul> <li>Employ forms of mathematical reasoning and proof appropriate to the solution of the problem, including deductive and inductive reasoning, making and testing conjectures and using counterexamples and indirect proof.</li> <li>Show mathematical reasoning in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models</li> </ul>

## Magic Squares

This problem gives you the chance to:

- work with magic squares, calculating cell values
- understand simple algebraic notation

6	1	8
7	5	3
2	9	4

In this square, adding the numbers in each row, each column and each diagonal gives the same result.

For such a square to be a "magic square" all nine numbers must be different.

1. Find the sum of each row, each column and each diagonal for this magic square.

x + z	x-y-z	x + y
x + y - z	x	x - y + z
x-y	x + y + z	x - z

This is the general form of a magic square, in which x, y and z represent numbers.

Find the sum of each row, each column and each diagonal for this square.

Here is a partially completed magic square.

3. Use algebra to complete this magic square.

12	14
4	6

4. Find the sum of each row, each column and each diagonal for the completed square. \_



Page 1

Magic Squares Test 9

Magic Squares					Ru	ıbric
The co • work • unde Based o	The core elements of performance required by this task are: • work with magic squares, calculating cell values • understand simple algebraic notation Based on these, credit for specific aspects of performance should be assigned as follows					
1.	1. Gives correct answer: 15					
2. Gives correct answer: <b>3x</b>					2	2
3.	All five values correct. <i>Partial credit</i>	12	1	14	2	
4 or 3 values correct <b>11 9 7</b>						
		4	17	6		
	Some evidence of the correct use of algebra.				2	4
4. Gives correct answer: 27			1	1		
			To	otal Points		8

Course 1 T	ask 2
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Student	Match graphic displays to the written descriptions of how some					
Task	students are paying for their summer vacations. Write a					
	formula that describes each of the matched relationships and					
	then write a possible description for a new vacation saving					
	formula.					
Core Idea	Understand patterns, relations, and functions.					
1	• Generalize patterns using explicitly defined functions					
Functions and	• Understand relations and functions and select, convert					
Relations	flexibly among, and use various representations for					
	them					
	• Analyze functions of one variable by investigating local					
	and global behavior including slopes as rates of					
	change intercepts and zeros					
Core Idea	Represent and analyze mathematical situations and					
3	structures using algebraic symbols.					
Algebraic	• Understand the meaning of equivalent forms of					
Properties and	expressions equations inequalities or relations					
Representations	<ul> <li>Use symbolic algebra to represent and explain</li> </ul>					
Representations	mathematical relationships					
	I las symbolis expressions to represent relationships					
	arising from various contexts					
	Approximate and interpret rates of shange from graphic					
	• Approximate and interpret rates of change from graphic					
Cana Idaa	and numeric data					
Core Idea	Employ forms of mathematical reasoning and proof					
	appropriate to the solution of the problem, including					
Mathematical	deductive and inductive reasoning, making and testing					
Reasoning	conjectures and using counterexamples and indirect proof.					
	• Use induction to make conjectures and use deductive					
	reasoning to prove conclusions					
	• Draw reasonable conclusions about a situation being					
	modeled					

Course One – 2005

### Vacations

This problem gives you the chance to:

analyze relationships using graphs and algebra

Here is some information about how some students are paying for their summer vacations.

- Carla: Her mom gave her \$100 in January and Carla has saved \$25 every month since, starting in February.
- Arnie: Arnie put \$150 in his piggy bank in January.
- Sue: Sue booked her vacation in January. She had \$250 in her piggy bank. Starting in February, she is paying \$50 each month to the travel company.
- Ben: Starting in February, Ben saves \$30 every month.

Here are some graphs illustrating these situations.

1. Match each person with a graph and explain how you decided.



Amount of money in dollars (s)he has		Amount of money in dollars (s)he has			
	Number of months since January			Number of mo since January	nths
Ν	lame:	Nam	e:		
R	eason.	Reas	on.		
_					
_					
2. In	these equations, \$A is the amount of mo A = 250 - 50n A = 30n A = 150	oney and <i>n</i> is the m	umber of months	since January	
a. b.	Find the person for each of these equ Write a formula for the fourth person	ations. I.			
	Carla				
	Amie				
	Sue				
	Ben				
3. W	rite a possible description for this form	ala: $A = 50n + 150$	)		
_					8
Copyright €	a 2005 by Mathematics Assessment	Page 3		Vacations	Test 9
	Service. All rights reserved.	-		pg 23	
Course	Onc = 2003			PE- 20	

Vacations	Ru	bric
The core elements of performance required by this task are: • analyze relationships using graphs and algebra		
Based on these, credit for specific aspects of performance should be assigned as follows	points	points
<ol> <li>For each graph: Gives the correct name and identifies one or more features of the graph or situation that distinguishes it from the others, such as the intercept or slop e.</li> </ol>		
Sue: graph goes downhill, showing that the amount of money in her piggy bank is decreasing. Intercept \$250: Slope < 0 or -\$50	1	
Arnie: graph is horizontal, showing the amount of money in his piggy bank stays the same. Intercept \$150: Slope constant or 0	1	
Ben: graph starts at 0 and goes up in steady steps. Intercept \$0: Slope > 0 \$30	1	4
Carla: graph starts above 0 and goes up in steady steps. Intercept \$100: Slope > 0 or \$25	1	
2. a. Carla: none of the given equations Arnie: $A = 150$ Sue: $A = 250 - 50n$ Ben: $A = 30n$		
All 3 correct	2	
Partial credit 2 correct	(1)	
b. G ives a correct formula for Carla: $A = 100 + 25n$	1	3
3. Gives a correct description such as:		
Student starts with \$150 and saves \$50 a month.	1	1
Total Points		8

# Course 1Task 3Multiples of Three

Student	Given a statement regarding multiples of three, test it to see if it				
Task	is true, find examples that match the statement and explain and				
	justify conclusions.				
Core Idea	Represent and analyze mathematical situations and				
3	structures using algebraic symbols.				
Algebraic	• Compare and contrast the properties of numbers and				
<b>Properties and</b>	number systems including real numbers				
Representations	• Use symbolic algebra to represent and explain				
	mathematical relationships				
	• Use symbolic expressions to represent relationships				
	arising from various contexts				
Core Idea	Employ forms of mathematical reasoning and proof				
2	appropriate to the solution of the problem, including				
Mathematical	deductive and inductive reasoning, making and testing				
Reasoning	conjectures and using counterexamples and indirect proof.				
	• Explain the logic inherent in a solution process				
	• Use induction to make conjectures and use deductive				
	reasoning to prove conclusions				
	<ul> <li>Draw reasonable conclusions about a situation being modeled</li> </ul>				

Course One – 2005

# **Multiples of Three**

This problem gives you the chance to:

- test statements to see if they are true
- find examples to match a description
- explain and justify your conclusions

#### If a number is a multiple of three, its digits add up to a multiple of three.

For example, 15 is a multiple of three  $(15 = 3 \times 5)$  and 1 + 5 = 6, which is a multiple of three.

Also, the number 255 is a multiple of three  $(255 = 3 \times 85)$ and 2 + 5 + 5 = 12, which is a multiple of three.

Use the above rule to test whether 4721 is a multiple of three or not and explain how you
figured it out.

2. Use the above rule to find a 5-digit multiple of three and explain how you know you are correct.

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s. End suffy, at job dou file and apres of indee job attrajs get interingte of indee.	3.	Zara says, "	'If you add	two multip	les of three	you always ge	et another mult	iple of three.'
---	----	--------------	-------------	------------	--------------	---------------	-----------------	-----------------

Is Zara contect?	Is Zara correct?	
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Explain how you decided.

4. Phil says, "If you add two multiples of three you always get a multiple of six."

Explain how you decided.

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Multiples of Three Test 9

MARS Tasks – High School Course 1

Multiples of Three	Ru	bric
<ul> <li>The core elements of performance required by this task are:</li> <li>test statements to see if they are true</li> <li>find examples to match a description</li> <li>explain and justify your conclusions</li> <li>Based on these, credit for specific aspects of performance should be assigned as follows</li> </ul>	points	section points
1. Gives correct answer: <b>4721 is not a multiple of 3</b>	1	
Gives an explanation such as: 4 + 7 + 2 + 1 = 14 14 is not a multiple of 3, so 4721 is not a multiple of 3	1 1	3
2. Writes down a five digit number such as 21,111		
Adds the digits: $2 + 1 + 1 + 1 + 1 = 6$	1	
States that since 6 is a multiple of 3	1	
21,111 is a multiple of 3	1	2
$(21,111 = 3 \times 7,037)$		2
3. Gives correct answer: Yes supported by a correct explanation such as:		
If two numbers are multiples of 3, then they can be written in the form 3x and 3y (where x and y are integers)	2	
3x + 3y = 3(x + y), which is a multiple of 3		2
4. Gives correct answer: No with one counterexample such as: $6 + 15 = 21$	1	1
Total Points		8

Course 1 Task 4	Scatter Diagram
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Student	Explain the information presented in a scatter plot of students' scores		
Task	on two tests. Evaluate statements made about the relationships		
	found from the data and revise the statements if necessary.		
Core Idea	Select and use appropriate statistical methods to analyze data		
5	and understand and apply basic concepts of probability.		
Data Analysis	• Understand the relationship between two sets of data		
	<ul> <li>(bivariate) and describe trends and shape of the plot including correlations (positive, negative, or no) and lines of best fit</li> <li>Make inferences based on the data and evaluate the validity of conclusions drawn</li> </ul>		
Core Idea	Employ forms of mathematical reasoning and proof appropriate		
2	to the solution of the problem, including deductive and inductive		
Mathematical	reasoning, making and testing conjectures and using		
Reasoning	counterexamples and indirect proof.		
	• Use induction to make conjectures and use deductive reasoning to prove conclusions		
	• Draw reasonable conclusions about a situation being modeled		

Course One – 2005

# Scatter Diagram

This problem gives you the chance to:

· discuss and understand a scatter plot of real data

A group of 66 students took two tests; Test A and Test B.

In the scatter diagram, each square represents one student and shows the scores that student got in the two tests.



- The mean score for Test A was 19 and the mean score for Test B was 16. Plot a point to show this on the scatter diagram.
- Draw a line of best fit on the scatter diagram. How can a line of best fit be used?

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Scatter Diagram Test 9

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3. Here are five statements about the scores shown on the scatter diagram.

If a statement is true check ( $\sqrt{}$ ) it.

If it is not true, write a correct statement.

Statement	Check (\/) or write correct statement
The scatter diagram shows positive correlation between the scores on Test A and the scores on Test B.	
The lowest score on Test A is lower than the lowest score for Test B.	
The range of scores on Test B is 25.	
The student with the highest score on Test A also has the highest score on Test B.	
The biggest difference between a student's scores on the two tests is 5.	

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Course One - 2005

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Scatter Diagram Test 9

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Sca	tter Diagram	Ru	bric
The core elements of performance required by this task are: • discuss and understand a scatter plot of real data			section
Based	on these, credit for specific aspects of performance should be assigned as follows	points	points
1.	Point correctly plotted	1	1
2.	Draws a line that best fits the data.	1	
	Gives a correct statement such as: A line of best fit can be used to estimate a students' score in one test if you know their score in the other.	1	2
3.	Correctly completes the table such as:		
	$\checkmark$	1	
	No. The lowest score on Test A (7) is greater than the lowest score on Test B (3).	1	
	$\checkmark$	1	
	No. The student with the highest score on Test A (30) does not have the highest score on Test B (28).	1	
	No. The biggest difference is more than 5 (accept 14).	1	5
	Total Points		8

<b>Fraction Sequences</b>

Student	Extend a sequence of fractions and compare the values. Make		
Task	conjectures about the patterns in the values of the terms as well as		
	their equivalent decimal values.		
Core Idea	Understand patterns, relations, and functions.		
1	<ul> <li>Generalize patterns using explicitly defined functions</li> </ul>		
Functions	Understand relations and functions and select, convert		
and Relations	flexibly among, and use various representations for them		
Core Idea	Employ forms of mathematical reasoning and proof appropriate		
2	to the solution of the problem, including deductive and inductive		
Mathematical	reasoning, making and testing conjectures and using		
Reasoning	counterexamples and indirect proof.		
	• Use induction to make conjectures and use deductive		
	reasoning to prove conclusions		
	• Draw reasonable conclusions about a situation being modeled		

Course One – 2005

## **Fraction Sequences**

This problem gives you the chance to:

- extend a given sequence of fractions
- calculate and compare decimal values

Bob writes a sequence of fractions.

In his sequence the next term after  $\frac{x}{y}$  is  $\frac{y+x}{y+2x}$ For example, the term after  $\frac{2}{3}$  is  $\frac{3+2}{3+4} = \frac{5}{7}$ 

Bob begins to write his sequence in a table.

		Decimal value
term # 1	$\frac{2}{3}$	0.66666666
term # 2	$\frac{5}{7}$	0.7142857
term # 3	$\frac{12}{17}$	0.705882
term # 4		
term # 5		
term # 6		

1. Calculate term # 4, term # 5, term # 6 in Bob's fraction sequence, and write them in the table.

2. What do you notice about the decimal values of these terms?

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Fraction Sequences Test 9

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Anna writes a different sequence of fractions.

For example, the term after  $\frac{2}{3}$  is  $\frac{3-2}{3+2} = \frac{1}{5}$ 

Anna begins to write her sequence in a table.

In her sequence the next term after  $\frac{x}{y}$  is  $\frac{y-x}{y+x}$ 

term # 1	$\frac{2}{3}$	0.66666666
term # 2	$\frac{1}{5}$	0.2
term # 3		
term # 4		
term # 5		
term # 6		

2

3. Calculate term # 3, term # 4, term # 5, term # 6 in Anna's fraction sequence and write them in the table.

Decimal value

4. What do you notice about the values of these terms?

	Dave 0
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Course One – 2005	

Fraction Sequences Test 9

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Fra	ction Sequences				Ru	bric
The co • exte • calc	ore elements of performance requents of a given sequence of fractions ulate and compare decimal value	uired by this t es	ask are:			
Based	on these, credit for specific aspects	of performance	e should b	be assigned as follows	points	section points
1.	Correctly completes table:			Decimal value		
		term # 1	2 3	0.66666666		
		term # 2	57	0.7142857		
		term # 3	12	0.705882		
		term # 4	29 41	0.707317		
		term # 5	70 99	0.707070		
		term # 6	$\frac{169}{239}$	0.7071129		
	6 correct values				3	
	<i>Partial credit</i> 5 or 4 correct values 3 or 2 correct values				(2) (1)	3
2.	Gives a correct statement su	ch as:				
	from term #2 on, the values	are around (	0.7.		1	1
	Accept: the values increase t	then decreas	se.			I
3.	Correctly completes table:			Decimal value		
		term # 1	$\frac{2}{3}$	0.66666666		
		term # 2	1 5	0.2		
		term # 3	$\frac{2}{3}$	0.66666666		
		term # 4	$\frac{1}{5}$	0.2		
		term # 5	$\frac{2}{3}$	0.66666666		
		term # 6	$\frac{1}{5}$	0.2		
	8 correct values					
	Partial credit					
	7, 6 or 5 correct values				(2)	
	4, 3 or 2 correct values				(1)	3
4. Gives a correct statement such as:						
values oscillate between 0.2 and 0.666				1	1	
				Total Poi	nts	8

Course One - 2005

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#### Algebra

Course One/Algebra	Task 1	Swimming Pool
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Student Task	Work with trapezoids, volume, rates and time graphs in the context of
	a swimming pool.
Core Idea 4	Understand measurable attributes of objects; and understand the
Geometry &	units, systems, and process of measurement.
Measurement	
Core Idea 3	• Approximate and interpret rates of change, from graphic and
Alg. Properties	numeric data.
&	
Representations	
Core Idea 1	• Analyze functions of one variable by investigating local and
Functions and	global behavior, including slopes as rates of change, intercepts
Relations	and zeros.

Based on teacher observation, this is what algebra students knew and were able to do:

 Students were able to convert from seconds to hours, but were unsure what to do with the decimal

Areas of difficulty for algebra students:

- Finding volume of an unfamiliar shape
- Composing/ decomposing a shape into familiar parts
- Confusing a state rate of water flow with a steady rise in the depth of the pool
- Confusing the shape of the pool with the shape of the graph
- Not recognizing that after 5 feet the depth would increase at a steady rate

### Swimming Pool

This problem gives you the chance to:

· work with trapezoids, rates and time graphs in a real context

This diagram shows a swimming pool.

The top of the swimming pool is a rectangle measuring 30 feet by 60 feet.

Two of the sides of the pool are trapezoids.

The water is 8 feet deep at the deep end and 3 feet deep at the shallow end.



 Find the volume of water in the pool. \_\_\_\_\_\_ cubic feet Show your calculations. \_\_\_\_\_\_

The volume of water in the pool is 74,250 gallons.

 A pump fills the pool at a rate of 1 gallon per second. How long, in hours and minutes, does it take to deliver 74,250 gallons of water into the pool?

		hours	minute	s
Show your calculations.				
Copyright © 2006 by Mathematics Assessment Resource Service. All rights reserved.	Page 1		Swimming Pool	Test 9

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3. (a) Which of these graphs best represents the depth of the water in the pool as it is filled at a steady rate of one gallon per second?

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Swimming Pool	Ru	bric
<ul> <li>The core elements of performance required by this task are:</li> <li>work with trapezoids, rates and time graphs in a real context</li> <li>Based on these, credit for specific aspects of performance should be assigned as follows</li> </ul>	points	section points
1. Gives correct answer: 9,900 cubic feet	1	
Shows correct calculation such as: $60 \ge \frac{(8+3)}{2} \ge 30$	1	
		2
2. Gives correct answer: 20 hours, 37.5 minutes	1	
Shows correct calculation: dividing 74,250 by $60 \ge 60 = 20.625$ hours	1 1	3
3. (a) Gives correct answer: Graph <b>B</b>	1	
<ul><li>(b) Gives correct explanation such as: At first the depth increases quickly, but then more slowly as the water moves up the slope. For the final 3 feet, the depth increases at a constant rate.</li></ul>	2	
Partial credit		
A partially correct explanation.	(1)	3
Total Points		8

#### Algebra

Course One/	Algebra	Task 2	Odd Sums
Student Task	Work with o conjectures a	odd, even and consecutive about consecutive number	numbers. Make and justify s.
Coro Idea 2	Employ for	ms of mathematical reas	oning and proof appropriate

Core Idea 2	Employ forms of mathematical reasoning and proof appropriate to		
Mathematical	the solution of the problem, including deductive and inductive		
Reasoning	reasoning, making and testing conjectures and using		
	counterexamples and indirect proof.		
	• Show mathematical reasoning in a variety of ways including		
	words, numbers, symbols, pictures, charts, graphs, tables,		
	diagrams, and models.		
	• Draw reasonable conclusions about a situation being modeled.		

Based on teacher observation, this is what algebra students knew and were able to do:

- Give examples to fit constraints using consecutive numbers
- Understand definitions of odd numbers, even numbers, and consecutive numbers
- Knew rules like an odd number plus and even number equals an odd number

#### Areas of difficulty for algebra students:

- Using algebra to make justifications
- Giving a process for finding the consecutive numbers rather than making a justification
- Noting the pattern of multiples of three in part four, rather than the more specific pattern of multiples of six
- Giving an explanation about the three consecutive numbers in part 4, instead of giving a rule for how to tell if the answer could be written as the sum of 3 consecutive numbers

### Odd Sums

This problem gives you the chance to:

- · work with odd, even and consecutive numbers
- make and explain justifications

The odd number 9 can be written as the sum of two consecutive whole numbers.

9 = 4 + 5

The odd number 47 can be written as the sum of two consecutive whole numbers.

47 = 23 + 24

 Show that the odd numbers 15 and 99 can be written as the sum of two consecutive whole numbers.



Explain why EVERY odd number can be written as the sum of two consecutive whole numbers.

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Even numbers cannot be written as the sum of two **consecutive** whole numbers. Some even numbers can be written as the sum of three **consecutive** numbers. For example: 24 = 7 + 8 + 9

Find three other examples of even numbers that can be written as the sum of THREE consecutive whole numbers.

 Explain how you can tell whether an even number can be written as the sum of three consecutive whole numbers.

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Odd Sums Test 9

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Odd Sums	Rι	ıbric
<ul><li>The core elements of performance required by this task are:</li><li>work with odd, even and consecutive numbers</li><li>make and explain justifications</li></ul>		section
Based on these, credit for specific aspects of performance should be assigned	ed as follows points	points
1. Gives correct answers: <b>7 + 8</b>	1	
49 + 50	1	2
2. Gives a correct explanation, perhaps involving a description sums can be made ('subtract 1 from the number, divided and that gives the smaller of the two consecutive numbers	on of how the the result by 2, s in the sum')	
Algebraically All odd numbers can be written in the form $2n + 1$ , where And $2n + 1 = n + (n + 1)$	n is an integer.	2
3. Gives three correct examples.	2	
Partial credit Gives two correct examples.	(1)	2
<ul> <li>4. Gives a correct explanation such as: (An even number that is) a multiple of 3 can be written as consecutive whole numbers. or A multiple of 6 can be written as the sum of three consecunumbers.</li> </ul>	the sum of three 2 ative whole 2	2
	Total Points	8

#### Algebra

Course One/	Algebra	Task 3	Patchwork Quilt
Student Task	Recognize and e	extend a number pat	tern for a geometric pattern. Express
	a rule using alge	bra. Use inverse op	erations to solve a problem.
Core Idea 1	Understand par	tterns, relations, a	nd functions.
Functions	Generali	ze patterns using ex	plicitly defined functions.
and Relations	<ul> <li>Understa</li> </ul>	and relations and fur	nctions and select, convert flexibly
	among, a	and use various repr	resentations for them.
	Recogniz	ze and generate equ	ivalent forms of simple algebraic
	expressio	ons and solve linear	equations.

Based on teacher observation, this is what algebra students knew and were able to do:

- Extend the pattern to five and explain how they got their answer, usually noting the growth rate of 5.
- Work backwards to find the number of black hexagons needed for 66 white hexagons.

Areas of difficulty for algebra students:

- Writing a rule or formula
- Understanding the difference between a recursive rule and a generalized rule
- Understanding that a variable is not the same as a label
- Understanding how the first term is different or how the constant effects the progression of the pattern
- Expressing ideas in symbolic notation
- Order of operations

### Patchwork Quilt

This problem gives you the chance to:

- recognize and extend a number pattern
- express a rule using algebra

Sam is making a border for a patchwork quilt.

She is sewing black and white regular hexagons together.



Sam makes a table to show the number of black and white hexagons she needs.

Number of	Number
black	of white
 hexagons	hexagons
1	6
2	11
3	16
4	21

1. How many white hexagons does Sam need for 6 black hexagons?

Explain how you figured it out.

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Patchwork Quilt Test 9

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2.	How many	black hexagons does Sam need for 66 white hexagons?	
		0	

Explain how you figured it out.

3.	Write a formula that will help you to find how many white hexagons (W) Sam needs
	for <i>n</i> black hexagons.

4. Use your formula to find how many white hexagons Sam needs for 77 black hexagons.

white hexagons

Show your work.

Page 6

Patchwork Quilt Test 9

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Patchwork Quilt		
<ul> <li>The core elements of performance required by this task are:</li> <li>recognize and extend a number pattern</li> <li>express a rule using algebra</li> <li>Based on these, credit for specific aspects of performance should be assigned as follows</li> </ul>	points	section points
1. Gives correct answer: <b>31</b>	1	
Gives a correct explanation such as: The first black hexagon needs 6 white hexagons and the other five black hexagons each need five white hexagons. $6 + 5 \ge 31$ (accept 21 + 10 = 31)	1	2
2. Gives correct answer: 13	1	
Gives a correct explanation such as: The first black hexagon needs 6 white hexagons and the other black hexagons each need five white hexagons.		
$ \begin{array}{c} 60 - 6 - 60 \\ 60 \div 5 = 12 \\ 12 + 1 = 13 \end{array} $	1	
		2
3. Gives correct answer: $W = 5n + 1$ or equivalent	2	
Partial credit Gives an expression such as 5n + 1	(1)	2
4. Gives correct answer: <b>386</b>	1	
Shows work such as: $W = 5 \times 77 + 1$	1	2
Total Points		

#### Algebra

Course One/Algebra 188K 4 Frinting Ficker	Course One/Algebra	Task 4	<b>Printing Tickets</b>
---	--------------------	--------	-------------------------

Student Task	Compare price plans using graphs and formulae. Use inequalities in a practical context of buying tickets		
Core Idea 3	Represent and analyze mathematical situations and structures		
Alg. Properties	using algebra.		
&	• Write equivalent forms of equations, inequalities and systems		
Representations	of equations and solve them		
	• Use symbolic algebra to represent and explain mathematical relationships		
	• Judge the meaning, utility, and reasonableness of results of		
	symbolic manipulations		

Based on teacher observation, this is what algebra students knew and were able to do:

- Write an equation for Best Print
- Draw a graph to match their equation
- Interpreting graphs of two equations to determine best buy under different conditions

Areas of difficulty for algebra students:

- Understand how to use symbolic notation to represent a context
- Find a table of values before drawing a graph
- Using algebra to solve for 2 equations with 2 unknowns

### **Printing Tickets**

This problem gives you the chance to:

· compare price plans using graphs and formulae

Susie is organizing the printing of tickets for a show.

She has collected prices from several printers and these two seem to be the best.



 Using C for the cost of the printing and t for the number of tickets, Susie writes a formula for each of the printers. Here is her formula for Sure Print:

Sure Print 
$$C = \frac{2t}{25}$$

Write the formula for Best Print:

Best Print C =



2. Susie's brother Rob has drawn *Sure Print*'s graph on a grid. Draw the graph for *Best Print*.

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Test 9

3. Susie uses algebra to find the values of C and t when the cost of printing the tickets is the same for both of the printers.

C = \_\_\_\_\_ t = \_\_\_\_

	Show how Susie may	have calculated C and t	i.		
4	What do Pak's graphs	and Sucia's calculation	as tall us shout the cost of t	ha tiakata?	
4.	Which company shoul	d Susie choose under w	what circumstances?	ne tickets?	
					9
Copy Reso	right © 2006 by Mathematics Assessment urce Service. All rights reserved.	Pag	e 8	Printing Tickets	Test 9

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Printing Tickets	Rubric	
<ul> <li>The core elements of performance required by this task are:</li> <li>compare price plans using graphs and formulae</li> <li>Based on these, credit for specific aspects of performance should be assigned as follows</li> </ul>	points	section points
1. Gives correct formula such as: $C = 10 + t / 25$	1	1
2. Draws a correct straight line from: (0, 10) to (400, 26)	2	2
3. Gives correct answers: $C = 20$ t = 250	1 1	
Shows correct work such as: $2t \div 25 = 10 + t \div 25$ 2t = 250 + t $C = 2 \ge 250 \div 25$	1	4
<ol> <li>Gives a correct explanation such as: If Susie buys less than 250 tickets, Sure Print will be cheaper, and if she buys more than 250 tickets, Best Print will be cheaper.</li> </ol>	1	2
Total Points		9
Course One/Algebra	Task 5	Graphs
--------------------	--------	--------
--------------------	--------	--------

Student Task	Relate line graphs to their equations.	
Core Idea 3 Alg.	Represent and analyze mathematical situations and structures	
Properties &	using algebraic symbols.	
Representations	• Understand the meaning of equivalent forms of expressions,	
	equations, inequalities, or relations	
	• Write equivalent forms of equations, inequalities, and systems	
	of equations and solve them	
Core Idea 1	• Analyze functions of one variable by investigating local and	
Functions and	global behavior, including slopes as rates of change, intercepts	
Relations	and zeros.	

Based on teacher observation, this is what algebra students knew and were able to do:

- Students could identify the origin
- Students could identify the equation x + y = 9
- Students could identify the equation y = 1/2 x
- Students could give the coordinates for the intersection of y=6 and x=6

- Confusing the order of x an y in a coordinate pair
- Recognizing the solutions for simultaneous equations
- Writing an equation for a line through a given point

### Graphs

This problem gives you the chance to:

· relate given line graphs to their equations

Here is a graphical diagram:



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Graphs Test 9

#### Algebra - 2006

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**Rubric** 1. Choose the correct label for each feature of the diagram from this list. Write its letter in the correct place in the diagram. (Not all the letters are needed.) А The line y = 0The line x = 0в С The line x = 6D The line y = 6Е The origin F The line  $y = \frac{1}{2}x$ G The line x + y = 9н The line y = x + 6L The line y = x - 6J The solution of the simultaneous equations x + y = 9 and  $y = \frac{1}{2}x$ κ The solution of the simultaneous equations x + y = 9 and y = 2x2. Which point is on the line y = 6 and on the line x = 6? 3. Write the equation of any straight line that goes through the point (3,6). Page 10 Copyright © 2006 by Mathematics Assessment Resource Service. All rights reserved.

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Gra	Graphs			
The core elements of performance required by this task are: • relate given line graphs to their equations Based on these, credit for specific aspects of performance should be assigned as follows		points	section points	
1.	Gives 7 correct labels (see below)	5		
	Partial credit			
	6 correct labels	(4)		
	5 correct labels	(3)		
	4 or 3 correct labels	(2)		
	2 or 1 correct label(s)	(1)	5	
2.	Gives correct answer: (6,6)	1		
			1	
3.	Gives any correct line, for example, $y = 6$ , $x = 3$ , $y = x + 3$ , $y = 9 - x$ , etc.	1		
			1	
	Total Points		7	



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Algebra	Task 1	Graphs	
Student Task	Work with linear and quadratic functions, their graph	s, and equations.	
Core Idea 1	Understand patterns, relations, and functions.		
<b>Functions and</b>	Analyze functions of one variable by investig	ating local and	
Relations	global behavior including slopes as rates of cl	nange, intercepts,	
	and zeros.		
Core Idea 3	Represent and analyze mathematical situations and structures		
Algebraic	using algebraic symbols.		
<b>Properties and</b>	• Understand the meaning of equivalent forms	of expressions,	
Representations	equations, inequalities, or relations.		
	Write equivalent forms of equations, inequali	ties and systems	
	of equations and solve them.		
	• Use symbolic algebra to represent and explain	n mathematical	
	relationships.		

### Mathematics in this task:

- Distinguish between linear and quadratic equations and their graphical representations
- Ability to graph a linear equation
- Ability to locate points on a graph and interpret their meaning
- Use algebra to find the intersections of two equations

Based on teacher observation, this is what algebra students knew and were able to do:

- Find the coordinates where the graphs intersect
- Give a reason for connecting equations with their graphs
- Draw a graph of y=3x

### Areas of difficulties for algebra students:

- Finding values for x, where one graph or equation is less than another
- Using algebra to find the points of intersection for two equations
- Knowing that the equations should equal each other at the points of intersection
- Using factoring as a tool to solve a quadratic equation
- Understanding that you can't divide by 0

### Strategies used by successful students:

- Making a table of values to help them graph
- Understanding y=mx+b and using it to help them graph
- Substitution

# Graphs

This problem gives you the chance to:

• work with linear and quadratic functions their graphs and equations



This diagram shows the graphs of  $y = x^2$  and y = 2x.

1. Fill in the labels to show which graph is which. Explain how you decided.

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2. Use the diagram to help you complete this statement:

2x is greater than  $x^2$  when x is between \_\_\_\_\_ and \_\_\_\_\_

- 3. The graphs of  $y = x^2$  and y = 2x cross each other at two points.
  - a. Write down the coordinates of these two points.
  - b. Show how you can use algebra to find the coordinates of the two points where the two graphs cross.

4.

- a. On the diagram, draw the graph of y = 3x.
- b. What are the coordinates of the points where  $y = x^2$  and y = 3x meet?
- c. Where do you think that the graphs of  $y = x^2$  and y = nx meet?
- d. Use algebra to prove your answer.

Graphs Test 9

Tas	sk 1: Graphs	Rı	ubric
The c • wo Based	core elements of performance required by this task are: ork with linear and quadratic functions their graphs and equations d on these, credit for specific aspects of performance should be assigned as follows	points	section points
1.	Graphs correctly labelled and convincing reason given	1	1
2.	Gives correct answer: between 0 and 2	1	1
3.a	Gives correct answer: (0, 0) and (2, 4)	1	
b	Shows correct reasoning to justify the answers in 3.a, such as: When the graphs meet,		
	$x^2 = 2x$ $\therefore x^2 - 2x = 0$	1	
	x(x-2) = 0		
	So $x = 0$ or $x = 2$		
	When $x = 0$ , $y = 0$ and when $x = 2$ , $y = 4$		
	So the coordinates are $(0, 0)$ and $(2, 4)$	1	3
4.a	Correct graph drawn	1	
b	Gives correct answer: (0, 0) and (3, 9)	1	
c	Gives correct answer: $(0, 0)$ and $(n, n^2)$	1	
d	Shows correct work such as: When the graphs meet, $x^2 = nx$		
	$\therefore x^2 - nx = 0$		
	x(x-n) = 0	1	
	So $x = 0$ or $x = n$		
	When $x = 0$ , $y = 0$ and when $x = n$ , $y = n^2$		
	So the coordinates are $(0, 0)$ and $(n, n^2)$		4
	Total Points		9

Student	Work with graphs and formulas in a real context.			
Task				
Core Idea 5	Select and use appropriate statistical methods to analyze data.			
Data Analysis	• Understand the relationship between two sets of data, display			
	such data in a scatterplot, and describe trends and shape of the			
	plot including correlations (positive, negative, and no) and			
	lines of best fit.			
	• Make inferences based on the data and evaluate the validity of			
	conclusions drawn.			
Core Idea 3	<b>Represent and analyze mathematical situations and structures</b>			
Algebraic	using algebraic symbols.			
<b>Properties and</b>	• Use symbolic expressions to represent relationships arising			
Representations	from various contexts.			
	• Approximate and interpret rates of change, from graphic and numeric data.			

### Mathematic in this task:

- Understanding information on a scatterplot, looking for trends such as correlation or no correlation
- Recognizing that a linear function passing through the origin is a proportion and finding a formula for a proportion
- Reading and interpreting points on a graph
- Graphing inequalities

Based on teacher observations, this is what algebra students knew and were able to do:

- Read and locate points on a scatterplot to meet constraints of the context
- Recognize when there is no pattern in a scatterplot
- Describe a trend in a scatterplot

### Areas of difficulty for algebra students:

- Finding a formula for a line on a graph
- Graphing an inequality on a graph from a verbal description

## **House Prices**

This problem gives you the chance to: • work with graphs and formulas in a real context

In March 2006, a newspaper article reported that houses in Maryland are so expensive that many people are unable to afford the monthly house payments.

This graph shows the average house price and the average monthly payment for all the different counties in Maryland.



**House Prices and Payments** 

1.

- a. What does the pattern of the data indicate about the connection between house prices and monthly payments?
- b. Find the monthly payment for a house costing \$450 000.
- c. Find a formula connecting the average monthly payment with the average house price.

House Prices Test 9

This graph shows the average monthly wage and the average monthly house payment for each county in Maryland.



#### 2.

a. Describe the pattern of the data.

b. Draw a ring round the point representing the county where the average person will find it most difficult to afford the monthly house payment. Label this point with the letter A.

c. Draw a ring round the point representing the county where the average person will find it easiest to afford the monthly house payment. Label this point with the letter B.

d. Indicate clearly which part of the graph contains points representing counties where the average monthly house payment is more than the average monthly wage.

House Prices Test 9

Tas	sk 2: House Prices	Ru	Ibric
<ul> <li>The core elements of performance required by this task are:</li> <li>work with graphs and formulas in a real context</li> <li>Based on these, credit for specific aspects of performance should be assigned as follows</li> </ul>		points	section points
1.a	Gives correct explanation such as:		
	There is a positive correlation between the two variables.	1	
b	Gives correct answer in the range \$3400 and \$3800	1	
c	Gives correct answer such as: $y = 0.008x$ (approximately) or $y = x/125$ , where $x$ is the house price and $y$ is the monthly payment or equivalent	1	2
	Accept an intercept in the range 0 to 100.		3
2.a	Gives correct explanation such as: No correlation or equivalent	1	
b	Point A correctly indicated: (2500, 4450)	1	
c	Point B correctly indicated: (2360, 800), or (3770, 1270)	1	
d	Clear indication of correct region, above the line $y = x$	1	4
	Total Points		7

### Ash's Puzzle Algebra

Student Task	Find numbers that obey given rules or constraints. Find rules for sets		
	of numbers. Use understanding of place value to solve problems in		
	context.		
Core Idea 3	Represent and analyze mathematical situations and structures		
Algebraic	using algebraic symbols.		
<b>Properties and</b>	• Use symbolic expressions to represent relationships		
Representations	arising from various contexts.		
	<ul> <li>Compare and contrast the properties of numbers and</li> </ul>		
	number systems including real numbers		
Core Idea 2	Employ forms of mathematical reasoning and proof appropriate		
Mathematical	to the solution of the problem, including deductive and inductive		
Reasoning	reasoning, making and testing conjectures and using		
	counterexamples and indirect proof.		
	• Show mathematical reasoning in a variety of ways, including		
	words, numbers, symbols, pictures, charts, graphs tables,		
	diagram and models.		
	• Explain the logic inherent in a solution process.		
	• Use induction to make conjectures and use deductive		
	reasoning to prove conclusions.		
	• Draw reasonable conclusions about a situation being modeled.		

Mathematics in this task:

- Investigating a relationship in number calculations
- Identifying relevant information using place value and number theory to discover a pattern in the solution
- Generalizing from arithmetic to a pattern for all solutions

Based on teacher observations, this is what algebra students knew and were able to do:

- Find most solutions which will reverse a two-digit number by adding nine
- Give examples of three-digit numbers that will reverse the digits when 99 is added and show supporting evidence

- Making an organized list or check for "all" solutions that meet a set of constraints
- Testing different cases of numbers, investigating enough options or choices before making a generalization
- Recognizing all the relevant information needed to make a convincing set of rules for all numbers
- Vocabulary for place value

## Ash's Puzzle

This problem gives you the chance to:

- find numbers that obey given rules
- find rules for sets of numbers

Ash has a book of number puzzles. This is one of the puzzles.



1. Solve this puzzle for Ash.

Show that your answer works.

Ash wonders if there are other answers to this puzzle.

2. Are there other correct answers to the puzzle?

If there are more correct answers list them all. If not explain how you know that there is only one correct answer.

Ash decides to try to find a three-digit number such that its digits are reversed when 99 is added. He finds that there are a lot of numbers that work.

3. Write **four** three-digit numbers that Ash could have found.

Show your work.

Ash thinks that there must be rules that would make it possible to find all of the three-digit numbers that are reversed when 99 is added to them.

4. Find these rules for Ash.

Task 3: Ash's Puzzle	Ru	bric
<ul><li>The core elements of performance required by this task are:</li><li>find numbers which obey given rules</li><li>find rules for sets of numbers</li></ul>	points	section points
Based on these, credit for specific aspects of performance should be assigned as follows		
1. Gives a correct answer: <b>12</b> , <b>23</b> , <b>34</b> , <b>45</b> , <b>56</b> , <b>67</b> , <b>78</b> or <b>89</b>		
and		
Gives correct calculation for their answer:		
such as $12 + 9 = 21$	1	
		1
2. Gives correct answer: yes		
and lists the other seven possible answers (ignore their answer to question 1 repeated) 12, 23, 34, 45, 56, 67, 78 and 89	2	
Partial credit:	(1)	
All extra 4, 5 of 6 correct answers with no incorrect ones.		2
3. Gives 4 correct answers:		
any 3 digit numbers with the last digit 1 greater than the first e.g. 152, 798 etc.	1	
Shows some correct work for their answers such as:	1	
152 + 99 = 251		2
4. Gives correct rules such as:		
The last digit is one more than the first.	1	
The middle digit can be any number.	1	
		2
Total Points		7

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Student Task	Form expressions and solve an equations to solve an age problem.	
Core Idea 3	Represent and analyze mathematical situations and structures	
Algebraic	using algebraic symbols.	
<b>Properties and</b>	• Use symbolic expressions to represent relationships	
Representations	arising from various contexts.	
	• Judge the meaning, utility, and reasonableness of results	
	of symbolic manipulations.	

### Mathematics in this task:

- Write algebraic expressions to represent the relationships between students' ages
- Understand how to use symbolic notation to express distributive property for a multiplicative relationship
- Solve an equation
- Reason about elapsed time to find when Jan's age is double Will's age
- Identify constraints and use them to set up an equation

Based on teacher observations this is what algebra students know and are able to do:

- Write an algebraic expression for an additive relationship
- Find the ages for the three children

### Areas of difficulty for algebra students:

- Writing a multiplicative expression that involves distributive property
- Writing and using an equation in a practical setting
- Understanding elapsed time and expressing elapsed time numerically or algebraically

## How Old Are They?

This problem gives you the chance to:

- form expressions
- form and solve an equation to solve an age problem

Will is *w* years old.

Ben is 3 years older.

1. Write an expression, in terms of *w*, for Ben's age.

Jan is twice as old as Ben.

2. Write an expression, in terms of *w*, for Jan's age.

If you add together the ages of Will, Ben and Jan the total comes to 41 years.

3. Form an equation and solve it to work out how old Will, Ben, and Jan are.

Will is \_\_\_\_\_ years old

Ben is years old

Jan is \_\_\_\_\_ years old

Show your work.

4. In how many years will Jan be twice as old as Will?	 years
Explain how you figured it out.	



Task 4: How Old Are They?		ubric
The core elements of performance required by this task are: • form expressions • form and solve an equation Based on these, credit for specific aspects of performance should be assigned as follows		section points
1. Gives a correct expression: $w + 3$	1	1
2. Gives a correct expression: 2(w + 3)	1	1
3 Gives correct answers: Will is 8 years old Ben is 11 and Jan is 22 years old	1	
Shows correct work such as: w + w + 3 + 2(w + 3) (allow follow through)	1	
$4_{W} + 9 = 41$ $4_{W} = 32$	1 ft.	3
4. Gives a correct answer: in 6 years time	1	
<ul> <li>Gives a correct explanation such as:</li> <li>Will is 14 years younger than Jan so when Will is 14 Jan will be 28.</li> <li>14 - 8 = 6.</li> <li>Accept guess and check with correct calculations.</li> <li>Solves correct equation.</li> </ul>	1	2
Total Point	s	7

Student Task	Find solutions to equations and inequalities.
Core Idea 3	Represent and analyze mathematical situations and structures
Algebraic	using algebraic symbols.
<b>Properties and</b>	• Write equivalent forms of equations, inequalities and systems
Representations	of equations and solve them.
	• Understand the meaning of equivalent forms of expressions,
	equations, inequalities, or relations.

Mathematics in this task:

- Ability to understand what is meant by the solution to an equation or inequality.
- Ability to calculate solutions to inequalities and equalities.
- Ability to use exponents, negative numbers and square roots.
- Ability to think about classes of numbers and the difference between finite and infinite sets
- Ability to recognize properties of equations and inequalities with regards to the number of possible solutions
- Ability to understand variable in a variety of ways

Based on teacher observations, this is what algebra students knew and were able to do:

- Find two solutions for  $1776x + 1066 \ge 365$
- Find both solutions for  $x^2=121$
- Find  $x^2 > x^3$  and for |x| > x

Areas of difficulty for algebra students:

- Understanding variable
- Understanding infinity
- Finding solutions for inequalities
- Identifying equations that have a limited number of solutions
- Thinking about classes of equations
- Working with exponents
- Understanding that the x's represent the same number with an equation

## **Two Solutions**

This problem gives you the chance to:

- · find solutions to equations and inequalities
- 1. For each of the following equalities and inequalities, find two values for x that make the statement true.
  - a.  $x^2 = 121$

b.  $x^2 = x$ 

- c.  $x^2 < x$
- d.  $(x-1)(5x^4 7x^3 + x) = 0$
- e.  $1776x + 1066 \ge 365$
- f.  $x^2 > x^3$
- g. |x| > x

2. Some of the equations and inequalities on the page opposite have exactly two solutions; others have more than two solutions.

a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

c. Write down two equations or inequalities that have an infinite number of solutions.

Task 5: Two Solutions	Rubric	
The core elements of performance required by this task are: • find solutions to equations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
<ol> <li>Gives correct answers:</li> <li>a: ± 11</li> <li>b: 0, 1</li> <li>c: any values between 0 and 1</li> <li>d: 0, 1</li> </ol>	7 x 1	
<ul> <li>e: any value ≥ -0.3947</li> <li>f: any value less than 1 except 0</li> <li>g: any negative value</li> </ul>		7
2. Gives correct answers with reasons such as:		
a. $x^2 = 121$ and $x^2 = x$ These are quadratic equations with two roots	1	
b. $(x-1)(5x^4-7x^3+x)=0$	1	
c. Gives two of:		
$x^2 < x, 1776x + 1066 \ge 365, x^2 > x^3,  x  > x$	1	3
Total Points		10

		<b>r</b>
Student Task	Work with algebraic e	xpressions for areas and perimeters of
	parallelograms and tra	pezoids.
Core Idea 3	<b>Represent and analy</b>	ze mathematical situations and structures
Algebraic	using algebraic symb	ools.
<b>Properties and</b>	• Use symbolic	algebra to represent and explain mathematical
Representations	relationships.	

Expressions

### Mathematics of this task:

Algebra

- Using variables to find area and perimeter of a parallelogram
- Recognizing equivalent expressions by factoring or using distributive property
- Using algebra to make and prove a generalization

Task 1

• Explaining the steps of factoring or using distributive property to make two expressions equivalent

Based on teacher observations, this is what algebra students know and are able to do:

- Students were able to recognize expressions for finding perimeter of a parallelogram
- Many students could recognize equivalent expressions for perimeter

- Finding the area of a parallelogram/ translating from words to variables (the formula for area was given in a verbal form)
- Thinking with variables instead of numbers
- Decomposing the trapezoid into two triangles
- Using factoring and/or distributive.0 property to make equivalent expressions
- Using algebra to make a generalization

### Expressions

This problem gives you the chance to:

 work with algebraic expressions for areas and perimeters of parallelograms and trapezoids



a. Which of these are correct expressions for the area of this parallelogram? Draw a circle around any that are correct.



- b. Which of these are correct expressions for the perimeter of the parallelogram? Draw a circle around any that are correct.
  - ab  $\frac{1}{2}ab$  ah  $\frac{1}{2}ah$  2a+2b 2(a+b) abh
- 2. Here is a trapezoid. It is made up of two triangles, each with height *h*.



Find the area of each of the two triangles and use your results to show that the area of the trapezoid is  $\frac{1}{2}(a+b)h$ .

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Expressions		
The core elements of performance required by this task are: • work with algebraic expressions for areas and perimeters of parallelograms and trapezoids		
Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1.a Gives correct answer: <i>ah</i> circled and no others circled	1	
b. Gives correct answers: $2a + 2b$ and $2(a + b)$	2 x 1	
Deduct 1 point for 1 extra and 2 points for more than 1 extra.		3
2. Provides a convincing development of the required expression such as:		
Shows the areas of the two triangles are $\frac{1}{2}ah$ and $\frac{1}{2}bh$	2 x 1	
Adds these two expressions to get $\frac{1}{2}(a+b)h$	1	3
Total Points		6

### Task 2

Student Task	Form and solve a pair of linear equations in a practical situation.	
Core Idea 3	Represent and analyze mathematical situations and structures	
Algebraic	using algebraic symbols.	
<b>Properties and</b>	• Use symbolic algebra to represent and explain mathematical	
Representations	relationships.	
	• Judge the meaning, utility, and reasonableness of results of symbolic manipulation.	

### Mathematics of the task:

Algebra

- Understanding equalities and maintaining equalities to set up and solve equations
- Defining variables
- Combining like terms, not combining unlike terms
- Understanding monetary units
- Understanding that two linear equations with two unknowns has a unique solution, but that a linear function with two variables has an infinite number of solutions
- Strategies or procedures for solving sets of equations: substitution, multiplication/addition

Based on teacher observations, this is what algebra students know and are able to do:

- Write an equation from context and understand what the variables represent
- Use guess and check successfully as a strategy to find a solution for two equations with two unknowns.
- Use substitution or multiplication/ addition to solve for the unknowns

- Quantifying answers to justify a conjecture
- Using distributive property correctly in solving a problem
- Identifying which equation and which unknown would be easiest to use when applying the substitution method
- Using partially remembered strategies, but could not carry them through the entire solution process or unsuccessfully combined strategies
- Checking work with both equations to see if the solution is true for both
- Understanding that functions have multiple solutions

# **Buying Chips and Candy**

This problem gives you the chance to: • form and solve a pair of linear equations in a practical situation

Ralph and Jody go to the shop to buy potato chips and candy bars.





Ralph buys 3 bags of potato chips and 4 candy bars. He spends \$3.75.

Jody buys 4 bags of potato chips and 2 candy bars. She spends \$3.00.

Later Clancy joins Ralph and Jody and asks to buy one bag of potato chips and one candy bar from them. They need to work out how much he should pay.

Ralph writes

3p + 4b = 375

1. If *p* stands for the cost, in cents, of a bag of potato chips and *b* stands for the cost, in cents, of a candy bar, what does the 375 in Ralph's equation mean?

2. Write a similar equation, using *p* and *b*, for the items Jody bought.

3. Use the two equations to figure out the price of a bag of potato chips and the price of a candy bar.

Potato chips\_\_\_\_\_

Candy bar \_\_\_\_\_

Show your work.

4. Clancy has just \$1. Does he have enough money to buy a bag of potato chips and a candy bar?

Explain your answer by showing your calculation.



Buying Chips and Candy	Ru	bric
The core elements of performance required by this task are: • form and solve a pair of linear equations in a practical situation		
Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
<ol> <li>Gives a correct explanation such as: It stands for the 375 cents that Ralph spent. (Must have correct units)</li> </ol>	1	1
2. Writes a correct equation such as: $4\mathbf{p} + 2\mathbf{b} = 300$	2	
<i>Partial credit</i> For an almost correct equation. (Left hand side of equation must be correct)	(1)	2
3. Gives correct answers: 45 cents or \$0.45 and 60 cents or \$0.60	1 ft	
Shows correct work such as: 8p + 4b = 6	110	
subtract $3p + 4b = 375$ 5p = 225 P = 45		
4 x 45 + 2b = 300 2b = 120 b = 60	2 ft	
Partial credit For some correct work.	(1 ft)	3
4. Gives a correct answer: no and		
Shows a correct calculation such as: 0.60 + 0.45 = 1.05	1 ft	
		1
Total Point	s	7

Student Task	Find relationships between graphs, equations, tables and rules.		
	Explain your reasons.		
Core Idea 1	Understand patterns, relations, and functions.		
Functions	• Understand relations and functions and select, convert flexibly		
and Relations	among, and use various representations for them.		
Core Idea 3	Represent and analyze mathematical situations and structures		
Algebraic	using algebraic symbols.		
<b>Properties and</b>	• Use symbolic algebra to represent and explain mathematical		
Representations	relationships.		
	• Judge the meaning, utility, and reasonableness of results of		
	symbolic manipulation.		

### The mathematics of this task:

- Making connections between different algebraic representations: graphs, equations, verbal rules, and tables
- Understanding how the equation determines the shape of the graph
- Developing a convincing argument using a variety of algebraic concepts
- Being able to move from specific solutions to thinking about generalizations

### Based on teacher observations, this is what algebra students know and are able to do:

- Understand that squaring a variable yields a parabola and that the variable that is squared effects the axis around which the parabola divided
- Use process of elimination as a strategy
- Match equations to tables and graphs
- Look for intercepts as a strategy
- Use vocabulary, such as: parabola, intercept, and linear

- Knowing the difference between linear and non-linear equations
- Not knowing how to explain how they matched the graph and the equation
- Connecting the constant to the slope, e.g. just because it's 2 doesn't meant it's negative slope
- Quantifying: even though they could describe the process, but didn't quantify
- Not knowing how or when to use the term "curve" or parabola

## **Sorting Functions**

This problem gives you the chance to:

- Find relationships between graphs, equations, tables and rules
- Explain your reasons

On the next page are four graphs, four equations, four tables, and four rules. Your task is to match each graph with an equation, a table and a rule.

1. Write your answers in the following table.

Graph	Equation	Table	Rule
Α			
В			
С			
D			

2. Explain how you matched each of the four graphs to its equation.

Graph A	 	 	 
Graph B	 		 
Graph C	 	 	 
Graph D _	 	 	 

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Sorting Functions					Rubric	
Th • Ba	ne core elements of perf find relationships betwo explain your reasons ased on these, credit for sp	ormance required by een graphs, equation pecific aspects of perform	y this task are: ns, tables and rules rmance should be as	s signed as follows	points	section points
1	Gives correct answe	rs:			2	
	Graph	Equation	Table	Rule	(2)	
	Α	С	В	Α	(1)	
	В	D	Α	С		
	С	В	С	D	3	
	D	Α	D	В	(2)	
					(1)	
	Equations: 4 correct 3 pc	oints Table: 4 corre	ct 3 points Rule:	4 correct 2	2	
	points	ainta 3 ar 2 correg	ot 2 points 3 or	2 correct 1	$(1)^{2}$	
	point	5 01 2 001100	<i>i z</i> politis 5 of		(1)	8
	1 correct 1 pc	oint 1 correc	et 1 point			
<ul> <li>2 Gives correct explanation such as:</li> <li>Graph A is a parabola/quadratic curve that passes through the origin and is symmetrical about the y axis (every value of y matches two values of x that are equal in size with opposite signs), so its equation is y = x<sup>2</sup>.</li> </ul>						
	<i>Graph C</i> is a parabo value of x matches t opposite signs), so it	la that is symmetri wo values of y that ts equation is $x = y$	cal about the x ax t are equal in size	- 2. tis (every with		
	<i>Graph D:</i> If we take ordinates, say, (2, 1) we have matched the	any point on the g , we get 2. This is e other three graph	raph and multiply the equation xy = s to equations.	y its co- 2. Accept,	2	
	Accept alternative correct explanations			(1)		
	Partial credit				(1)	2
	2 or 3 correct explan	ations				2 10
		Total Poin	nts			10

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### Task 4

### **Sidewalk Patterns**

Student Task	Work with patterns. Work out the nth term of a sequence.
Core Idea 1	Understand patterns, relations, and functions.
Functions and	• Understand relations and functions and select, convert flexibly
Relations	among, and use various representations for them.
Core Idea 3	Represent and analyze mathematical situations and structures
Algebraic	using algebraic symbols.
<b>Properties and</b>	• Use symbolic algebra to represent and explain mathematical
Representations	relationships.
	• Judge the meaning, utility, and reasonableness of results of
	symbolic manipulation.

Mathematics of the task:

Algebra

- Drawing and extending a visual pattern
- Using a table to extend a pattern
- Noticing relationships and number patterns, such as perfect squares
- Writing an expression to give the nth term in a sequence
- Working backwards, being able to "do and undo" a computational procedure

Based on teacher observations, this is what algebra students know and are able to do:

- Recognize squares, square numbers
- Draw and extend a visual pattern
- Extend a pattern by completing a table
- Compare the relationship between white and gray blocks by making observations from the table
- Find square roots of numbers

- Writing an equation or making a generalized rule
- Considering all the relationships in the pattern
- How to algebraically express an even number
- How to define the "added" number
- How to explain the difference in the blocks given the total, what to do with the 1/2
# **Sidewalk Patterns**

This problem gives you the chance to:

- work with patterns
  work out the n<sup>th</sup> term of a sequence

In Prague some sidewalks are made of small square blocks of stone.

The blocks are in different shades to make patterns that are in various sizes.

Pattern number 1

 	_	 _				
Do	tta	 	1110	ah	~	2





1. Draw the next pattern in this series.

You may not need to use all of the squares on this grid.

Pattern number 4

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2. Complete the table below.

Pattern number, n	1	2	3	4
Number of white blocks	12	40		
Number of gray blocks	13			
Total number of blocks	25			

- 3. What do you notice about the number of white blocks and the number of gray blocks?
- 4. The total number of blocks can be found by squaring the number of blocks along one side of the pattern.

a. Fill in the blank spaces in this list.

 $25 = 5^2$  81 = 169 =  $289 = 17^2$ 

b. How many blocks will pattern number 5 need?

c. How many blocks will pattern number *n* need?

- 5. a. If you know the total number of blocks in a pattern you can work out the number of white blocks in it. Explain how you can do this.
  - b. Pattern number 6 has a total of 625 blocks. How many white blocks are needed for pattern number 6? Show how you figured this out.

Sid	lewalk Patterns							Ru	bric
The o • wor • wor Based	core elements of performance re k with patterns k out the n <sup>th</sup> term of a sequence d on these, credit for specific aspec	equired by this task are e sts of performance should	e: I be ass	igned a	as follov	ws		points	section points
1.	Draws correct pattern:							1	1
2.	Gives correct answers:						7		
		Pattern number, n	1	2	3	4			
		Number of white blocks	12	40	84	144			
		Number of gray blocks	13	41	85	145			
		Total number of blocks	25	81	169	289		2	
	Partial credit: 5, 6 or 7 corr	rect						(1)	2
3.	Gives a correct answer such	as: there is one mo	re gra	y than	white	e blocł	KS	1	1
4.a.	Gives correct answers: $9^2$ a	and 13 <sup>2</sup>						1	
b	Gives correct answer: $21^2$	and/or 441						1	
		. 1)2							
c.	Gives correct answer: (4n	+1) <sup>-</sup> or equivalent						2	
									4
5.a. b.	Gives a correct answer such and Gives correct answer: <b>312</b> and	n as: Subtract 1 from	the to	otal an	d divi	de by	2.		
	Shows work such as: (625	(-1)/2							1
		, -			т	otol D	ointa		1
					1	otal P	omus		7

Algebra

**Functions** 

Student Task	Work with graphs and equations of linear and non-linear functions.
Core Idea 1	Understand patterns, relations, and functions.
Functions and	• Understand relations and functions and select, convert flexibly
Relations	among, and use various representations for them.
Core Idea 3	Represent and analyze mathematical situations and structures
Algebraic	using algebraic symbols.
<b>Properties and</b>	• Use symbolic algebra to represent and explain mathematical
Representations	relationships.
	• Judge the meaning, utility, and reasonableness of results of
	symbolic manipulation.

The mathematics of this task:

- Identify linear points on a coordinate grid and name them
- Write an equation for a linear function from a graph or from coordinates
- Recognize non-linear points that form a parabola and estimate the graph of the curve
- Distinguish between features of a linear, quadratic and exponential graph and their equations
- Find the equation for a parabola given some of the coordinate points

# Based on teacher observations, this is what algebra students knew and were able to do:

- Understand that a linear graph is a straight line
- Know that a non-linear graph is a parabola
- Identifying points on a graph

### Areas of difficulty for algebra students:

- Finding a linear equation from a graph
- Finding a quadratic equation
- Drawing a parabola
- Difficulty in knowing difference between quadratic and quadrilateral (four points/four sides)
- Confusion about quadratic and exponential equations(x<sup>2</sup> has an exponent so it was exponential)

Strategies used by successful students:

- Knowing the generic formula for a line: y = mx + b and using slope and substitution
- Looking for the y-intercept for the linear equation
- Finding slope
- Knowing the generic formula for a quadratic:  $y=a x^2+bx + c$
- Knowing mathematical vocabulary: quadratic, exponential

# Functions

This problem gives you the chance to:

· work with graphs and equations of linear and non-linear functions

On the grid are eight points from two different functions.



Show your work.

For the **non-linear** function:

3. Write the coordinate pairs of its four points.

Algebra – 2008 Copyright © 2008 by Mathematics Assessment Resource Service All rights reserved. Draw the graph of the function on the grid.

4.	The non-linear function is quadratic	The non-linear function is exponential	
Chris			Alex
Who is correct?			
Explain your reason	18.		

5. Write an equation that fits the non-linear function.

Show your work.

Functions		Ru	bric
The core elements of performance required be • work with graphs and equations of linear and Based on these, credit for specific aspects of performance of the second sec	by this task are: nd non-linear functions ormance should be assigned as follows	points	section points
1. Gives correct answers: (2, 9), (3, 7) and Draws a correct line on the grid.	, (4, 5), (5, 3)	1	1
2. Gives correct answer: $y = 13 - 2x$		2	2
3. Gives correct answers: (1, 5), (2, 8	), (3, 9), (4, 8)	1	
Draws a correct curved graph or ec	quivalent	1	2
<ul> <li>Gives correct answer: Chris and</li> <li>Gives a correct explanation such as The graph has a turning point. or I</li> </ul>	: t is part of a parabola.	1	1
5. Gives correct answer: $y = 6x - x^2 c$	or equivalent such as $-(x-3)^2 + 9$	1	
Shows some correct work such as: Substitutes coordinates in $y = ax^2 + bx^2$	bx + c	1	2
	Total Points		8

Algebra

Student Task	Make an equation and solve a problem.
Core Idea 3	Represent and analyze mathematical situations and structures
Algebraic	using algebraic symbols.
Properties and	• Understand the meaning of equivalent forms of expressions,
Representations	equations, inequalities, or relations.
	• Write equivalent forms of equations, inequalities and systems
	of equations and solve them.
	• Use symbolic algebra to represent and explain mathematical
	relationships.

#### Mathematics of this task:

- Writing an expression from a diagram
- Understanding the relationship of equality in the context of a balanced scale
- Using equivalent ratios in context
- Distinguishing between a numerical coefficient and a value for the variable
- Understanding when there is sufficient information to solve for a variable
- Understanding meaning of variable or unknown

Based on teacher observations, this is what algebra students know and are able to do:

- Write expressions for diagrams
- Write equations for the diagram

### Areas of difficulties for algebra students:

- Trying to assign values to x and y
- Trying to combine like terms from different sides of an equality
- Understanding the concept of equality illustrated in the diagram
- Using relational thinking to find equivalent ratios

# Soup and Beans

This problem gives you the chance to:

• make an equation and solve a problem

The weight of one can of beans is x ounces.

The weight of one can of soup is y ounces.

- 1. Write an expression for the weight of the cans on the left hand side of the weighing scales.
- 2. Write an expression for the weight of the cans on the right hand side of the weighing scales.



3. Write an equation that shows the relationship between x and y.

4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

Show your work.



6

4

Soup and Beans

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# 2009 Rubrics

Soup and Beans	Ru	bric
<ul> <li>The core elements of performance required by this task are:</li> <li>make an equation and solve a problem</li> <li>Based on these, credit for specific aspects of performance should be assigned as follows</li> </ul>	points	section points
1. Gives correct answer: $5x + 2y$	1	
		1
2. Gives correct answer: $3x + 5y$	1	
		1
3. Gives correct answers: $5x + 2y = 3x + 5y$ or $2x = 3y$	1	
		1
4. Gives correct answers: 6	2	
Shows correct work such as:		
$2\mathbf{x} = 3\mathbf{y}$	1	
6x = 9y		3
Total Points		6

### Algebra

Student Task	Work with a quadratic function in various forms.
Core Idea 3	Represent and analyze mathematical situations and structures
Algebraic	using algebraic symbols.
<b>Properties and</b>	• Approximate and interpret rates of change, from graphic and
Representations	numeric data.
Core Idea 1	• Understand relations and functions and select, convert flexibly
<b>Functions and</b>	among, and use various representations for them.
Relations	

### Mathematics of the task:

- Calculations with exponents and negative numbers
- Codifying calculations into a symbolic string
- Rectifying two forms of an equation
- Interpreting graphical representations of linear and quadratic equations and identifying minimum point and solutions
- Using algebra to find the solution to a quadratic equation (using factoring or the quadratic equation)

Based on teacher observation, this is what algebra students know and are able to do:

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- Calculate using the number machine
- Identify the minimum point on a parabola using a graph

#### Areas of difficulty for algebra students:

- Using algebra to show that two equations are equal
- Finding solutions to two equations on a graph
- Using algebra to find the solutions to a quadratic equation

Quadratic

Task 2

## Quadratic

This problem gives you the chance to:

• work with a quadratic function in various forms

This is a quadratic number machine.



The diagram on the next page shows the graph of the machine's quadratic function  $y = x^2 - 2x - 8$ and the graphs of y = 3 and y = x.

2. a. Which point on the diagram shows the minimum value of *y*?

b. Which point(s) on the diagram show(s) the solution(s) to the equation  $3 = x^2 - 2x - 8$ ?

c. Which point(s) on the diagram show(s) the solution(s) to the equation  $x = x^2 - 2x - 8$ ?

3. a. Use the graph to solve the equation  $x^2 - 2x - 8 = 0$ . Mark the solutions on the graph.

x = \_\_\_\_\_ or x = \_\_\_\_\_

b. Use algebra to solve the same equation.



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Quadratic

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Quadratic	Ru	bric
<ul> <li>The core elements of performance required by this task are:</li> <li>work with a quadratic function in various forms</li> <li>Based on these, credit for specific aspects of performance should be assigned as follows</li> </ul>	points	section points
<ol> <li>a. Gives a correct answer: 5→4→16→7</li> <li>b. Gives a correct answer: -8</li> </ol>	1 1	
c. Gives a correct answer: $y = (x - 1)^{2} - 9$ $= x^{2} - 2x + 1 - 9$ $= x^{2} - 2x - 8$	2	4
2. a. Gives a correct answer: C	1	
b. Gives a correct answer: A and E	1	
c. Gives a correct answer: <b>B</b> and <b>D</b>	1	3
3. a. Gives a correct answer: -2, 4	1	
b. Gives a correct answer such as: $(x+2)(x-4) = 0$ , so $x = -2$ or 4 or uses completing the square to find 1 or 2 correct answers or uses quadratic formula.	1	2
Total Points		9

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## Algebra

Student Task	Find numbers that obey given rules or constraints. Find rules for sets
	of numbers. Use understanding of place value to solve problems in
	context.
Core Idea 3	Represent and analyze mathematical situations and structures
Algebraic	using algebraic symbols.
<b>Properties and</b>	<ul> <li>Use symbolic expressions to represent relationships</li> </ul>
Representations	arising from various contexts.
	• Compare and contrast the properties of numbers and
	number systems including real numbers
Core Idea 2	Employ forms of mathematical reasoning and proof appropriate
Mathematical	to the solution of the problem, including deductive and inductive
Reasoning	reasoning, making and testing conjectures and using
	counterexamples and indirect proof.
	• Show mathematical reasoning in a variety of ways, including
	words, numbers, symbols, pictures, charts, graphs tables,
	diagram and models.
	• Draw reasonable conclusions about a situation being modeled.
Core Idea 1	• Understand relations and functions and select, convert flexibly
<b>Functions and</b>	among, and use various representations for them.
Relations	

Mathematics of the task:

- Know and be able to calculate volume, area, and circumference
- Reason about shapes of graphs, particularly relative to linear functions and quadratic or cubic functions
- Solve equations for different variables

Based on teacher observations, this is what algebra students knew and were able to do:

- Recognize that volume would show the steepest growth curve
- Match the graphs to the descriptions
- Solve area formula for the radius

### Areas of difficulty for algebra students:

- Explaining why the descriptions matched the graphs
- Giving quantities for some measures, but not for all to make the comparison
- Solving a formula for a different variable

# **Circles and Spheres**

This problem gives you the chance to:

- · compare functions for length, area and volume
- rearrange formulas

1. The diagram shows the graphs of three functions:

- Area of circle against radius
- Volume of sphere against radius
- Circumference of circle against radius

In each case, the radius varies from 0 to 5 units,

A A B C C Radius

For each letter, choose the correct description of the graph from the list above.

Α_	
Ъ	
в_	
С	
_	

Say how you figured it out.

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The formula to calculate the volume of a sphere from its radius is

 $V = \frac{4}{3}\pi r^{3}$ 

where r is the radius and

V is the Volume

2. The formula to calculate the area of a circle from its radius is  $A = \pi r^2$ , where *r* units is the radius and *A* square units is the area.

Which of the formulas below works out the radius of a circle from its area?

Say 'yes' or 'no' for each one.



3. The formula to calculate the circumference of a circle from its radius is  $C = 2\pi r$ , where r units is the radius and C units is the circumference.

Find a formula to work out the radius of a circle from its circumference.



Circles and Spheres 52

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Circles and Spheres			Rubric				
The • c • re	core elements of p ompare functions fo earrange formulas	erformance requ or length, area a	ired by this ta nd volume	sk are:	F	noints	section
Base	d on these, credit for	specific aspects o	f performance	should be assigned as follo	WS	points	points
1.	Gives correct a	nswers:					
	A: Volume of s	sphere against	radius				
	B: Area of circ	le against radi	us			3x1	
	C: Circumfere	nce of circle ag	gainst radius	S			
	Gives correct ex C shows a linea	xplanations suc r relationship s	h as: o represents	length			
	B and A must represent the quadratic relationship for area and the cubic relationship for volume – comparing them shows B is the quadratic and C is the cubic graph				e cubic fic and C is	2	
	May calculate s	ome values suc	h as:			or	
	radius	volume	area	circumference			
	3	113	28	19			
	4	268	50	25		2	
	5	523	79	31			
	Partial credit					(1)	
	1 error					(1)	5
2.	Gives correct an	nswers: <b>No, no</b> ,	, no, yes			2	3
	Partial credit						
	One error					(1)	2
3.	Gives correct an	nswer: $r = \frac{C}{2\pi}$				1	
							1
				1	<b>'otal Points</b>		8

### **Words and Equations**

Algebra

Student Task	Form equations that match situations expressed in words.			
Core Idea 3	Represent and analyze mathematical situations and structures			
Algebraic	using algebraic symbols.			
<b>Properties and</b>	• Use symbolic expressions to represent relationships			
Representations	arising from various contexts.			
	• Judge the meaning, utility, and reasonableness of results			
	of symbolic manipulations.			
	• Understand the meaning of equivalent forms of			
	expressions, equations, inequalities or relations.			

### Mathematics of this task:

- Use symbolic expressions to describe relationships from verbal descriptions
- Use inverse operations in context
- Write equations and navigate between different representations
- Understand and choose appropriate operations to match a context

Based on teacher observations, this is what algebra students know and are able to do:

- Work with operations of addition and subtraction in context
- Manipulate equations to find equivalents

Areas of difficulty for algebra students:

- Forgetting negative sign when moving variable to a different side of an equation (working with inverse operations)
- Understanding order in a subtractions context (What is the whole? What is the part?)
- Choosing multiplication and division in context
- Understanding order in a division context (What is the quantity being divided? What is it being divided by?)

# Words and Equations

This problem gives you the chance to:

• write equations that match situations expressed in words

On this page are five situations expressed in words.

On the next page are ten algebraic equations.

Your task is to match each word situation with two algebraic equations.

Word situation	Matches two equations
<b>Cousins</b> Maria is 12 years older than her cousin Andy.	Equation 1
Maria is $x$ years old. Andy is $y$ years old.	Equation 2
<b>Walking</b> Tom is walking 12 miles from A to B	Equation 1
He has already walked <i>x</i> miles. He needs to walk y miles more.	Equation 2
<b>Boxes and bottles</b> There are twelve bottles in each box.	Equation 1
The total number of bottles is x. The number of boxes is y.	Equation 2
<b>Temperatures</b> It is 12° colder in the mountain than in the valley.	Equation 1
The temperature on the mountain is $x^{\circ}$ The temperature in the valley is $y^{\circ}$ .	Equation 2
<b>Years and months</b> There are twelve months in every year.	Equation 1
The number of years is x. The number of months is y.	Equation 2

$$x = 12y$$
 $y = x + 12$ 
 $y = x - 12$ 
 $x = y + 12$ 
 $x = y - 12$ 
 $y = 12x$ 
 $y = 12 - x$ 
 $y = \frac{x}{12}$ 
 $x = \frac{y}{12}$ 
 $x = 12 - y$ 



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Words and Equations		
The core elements of performance required by this task are: • write equations that match situations expressed in words Based on these, credit for specific aspects of performance should be assigned as follows		
1. Gives correct answer: $\mathbf{x} = \mathbf{y} + 12$ and $\mathbf{y} = \mathbf{x} - 12$	2 x 1	
Accept: $x - y = 12$ or equivalent answers.		2
2. Gives correct answer: $\mathbf{x} = 12 - \mathbf{y}$ and $\mathbf{y} = 12 - \mathbf{x}$	2 x1	
Accept: $x + y = 12$ or equivalent answers.		2
3. Gives correct answer: $x = 12y$ and $y = x/12$	2 x1	
Accept: $x/y=12$ or equivalent answers.		2
4. Gives correct answer: $\mathbf{x} = \mathbf{y} - 12$ and $\mathbf{y} = \mathbf{x} + 12$	2 x 1	2
Accept: $y-x=12$ or equivalent answers.		2
5. Gives correct answer: $x = y/12$ and $y = 12x$	2 x 1	2
Accept: $y/x=12$ or equivalent answers.		2
Total Point	5	10

# Algebra Task 5

Student Task	Use a chart to solve simultaneous equations.			
Core Idea 3	Represent and analyze mathematical situations and structures			
Algebraic	using algebraic symbols.			
Properties and	• Write equivalent forms of equations, inequalities and systems			
Representations	of equations and solve them.			
	• Understand the meaning of equivalent forms of expressions,			
	equations, inequalities, or relations.			

Coffee

### Mathematics of this task:

- Read and interpret a diagram
- Write equations from a graphical representation
- Solve 2 equations with 2 unknowns to find a unit price
- Use proportional reasoning or equations to find the cost of an amount not on the table

Based on teacher observations, this is what algebra students knew and were able to do:

- Read and interpret the table to describe the 500 using appropriate units
- Find the cost in part 2
- Use guess and check to find the unit prices in part 2

### Areas of difficulty for algebra students:

- Using the information in the table to write equations
- Solving simultaneous equations or recognizing the using equations could be a strategy to find the unit prices
- Trying to impose a unit of scale on the table without considering all the information

# Coffee

This problem gives you the chance to:

• use a chart to solve simultaneous equations

This chart shows the cost, in cents. of different numbers of small and large cups of coffee.



1. Explain what the number 500 in the chart means.

2. Use the information in the chart to find the cost of a small cup of coffee and the cost of a large cup of coffee. Show how you figured it out.

Small cup of coffee costs \_\_\_\_\_ cents Large cup of coffee costs \_\_\_\_\_ cents

3. What number should go in the empty box that the arrow is pointing to. \_\_\_\_\_\_ Explain your work.

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Coffee	Ru	bric
The core elements of performance required by this task are: • use a chart to solve simultaneous equations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
<ol> <li>Gives correct answer such as: the cost of 3 small cups of coffee and 2 large cups of coffee is 500¢.</li> </ol>	1	1
<ul> <li>Gives correct answer: small 80¢, large 130¢</li> <li>Shows work such as: solving simultaneous equations</li> <li><i>Partial credit</i></li> <li>Some correct work</li> </ul>	2 x 1 2 (1)	4
<ul> <li>Gives correct answer: 340 <ul> <li>and</li> <li>Gives correct explanation such as:</li> <li>The cost of one small cup and two large cups. (is half the cost of two small cups and four large cups.)</li> </ul> </li> </ul>	2ft	2
Total Points		7