MARS Tasks | Course 2

Page	Name of MARS Task	Year	Math Strand	Notes
*	Rectangles w/Fixed Area	2004	GM, PFA	Length & height in fixed areas
*	At the Gym	2004	GM	Work with radius, area, volume
*	Birds' Eggs	2004	DA, PFA	Interpret scatter diagram, compare gradients
*	Pentagons	2004	GM	Solve angle equations in pentagons
*	Differences	2004	PFA	Use algebra to explore sequences
2	Pipes	2005	GM, PFA	Use Pythagorean rule to calculate distance
5	Multiplying Cells	2005	PFA	Work with number sequence, power of 2
9	Sum of Two Squares	2005	PFA	Find pattern, make algebraic transformation
13	Quadrilaterals	2005	GM	Use geometric properties to solve problem
16	Ramps	2005	GM	Use trigonometry
19	London Eye	2006	GM	Find length/speeds of rotating wheel
23	Hopewell Geometry	2006	GM	Pythagorean Rule, angles and similarity
27	Quadratic	2006	PFA	Find properties of quadratic function
31	How Compact?	2006	GM, PFA	Use formula, calculate area, Pythagorean
35	Rhombuses	2006	GM	Properties, Pythagorean Rule, similarity
39	Insane Propane	2007	GM	Volume in cylinders and spheres
43	Sloping Squares	2007	GM, PFA	Calculate areas, use algebraic expressions
47	Sum and Product	2007	PFA	Use algebra, solve quadratic equation
51	Circles in Triangles	2007	GM, PFA	Use algebra in geometric situations
54	Two Solutions	2007	PFA	Find solutions to equations, inequalities
58	Numbers	2008	PFA	Prove number relations using algebra
61	Glasses	2008	GM	Calculate volumes of cylinders, cones
65	Pentagon	2008	GM	Demonstrate geometrical reasoning
69	Triangles	2008	GM	Areas of triangles within triangles
73	Circle and Square	2008	GM	Calculate areas, nested circles & squares
76	Triangles	2009	GM	Explain reasoning in problem situation
79	Hanging Baskets	2009	GM, PFA	Volumes of pyramids, hemispheres
83	Pentagon	2009	GM	Geometry and mathematical proofs
87	Circle Pattern	2009	GM, PFA	Algebraic patterns in geometric situation
91	Floor Pattern	2009	GM	Geometry and geometric reasoning

NP=Number Properties
NO=Number Operations
PFA=Patterns Functions Algebra
GM=Geometry & Measurement
DA=Data Analysis

* Tasks from 2004 are not included in this packet due to copyright restrictions. However, if you click on the name of the task, you can access it via the Noyce Foundation website. Tasks from 2005 to 2009 are available here with permission from the Mathematics Assessment Resource Service (MARS).

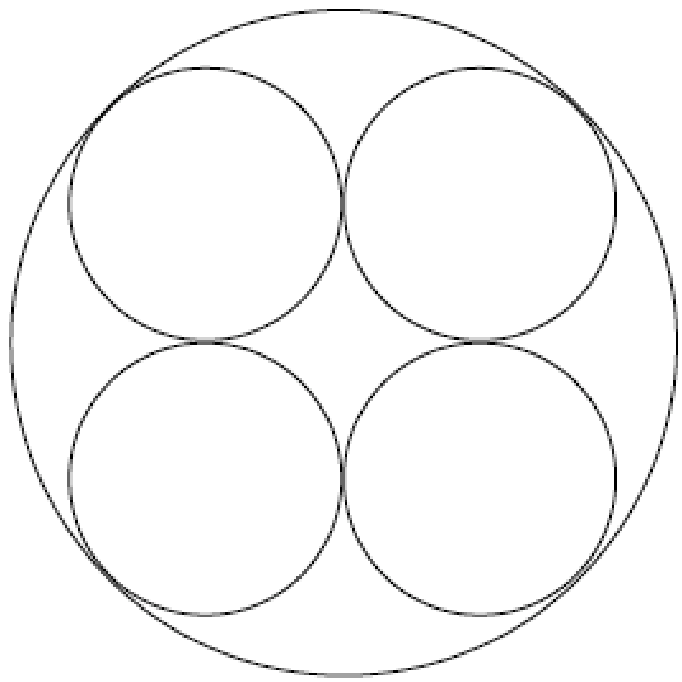
Course 2 Task 1 Pipes

Student Task	The task asks students to use Pythagorean Theorem to find the radius of a large pipe with 4 smaller pipes fitting inside the larger pipe.
Core Idea 3 Algebra Properties and Representations	• **Represent and analyze mathematical situations and structures using algebraic symbols..** • Solve equations involving radicals and exponents in contextualized problems such as use of Pythagorean theorem.
Core Idea 3 Geometry and Measurement	**Analyze characteristics and properties of two-dimensional geometric shapes; develop mathematical arguments about geometric relationships; and apply appropriate techniques, tools, and formulas to determine measurements.** • Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools. • Visualize three-dimensional objects from different perspectives and analyze their cross sections

Course Two – 2005

Pipes

This problem gives you the chance to:
- use the Pythagorean Rule
- calculate distances between circles



Four small circular pipes are placed inside a large circular pipe as shown above.
Each of the four small pipes has a radius of 6 inches.

Find the radius of the large pipe. Show how you figured it out.

(Hint: Join the centers of the 4 small circles and find the length of the diagonal of the square obtained.)

6

Course Two – 2005

Pipes		Rubric
The core elements of performance required by this task are: • use the Pythagorean Rule • calculate distances between circles Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
Gives a correct answer: 14.49 inches (approx. **14.5**) (Accept 14.4 to 14.5)	1	
Shows correct work such as:		
Draws correct diagram (as suggested in "hint")	1	
Shows that the sides of the square measure **12** inches (twice radius)	1	
Attempts to use Pythagorean Theorem.	1	
Shows correct work such as:		
The diagonals of the square measure $12\sqrt{2} = 16.9$ inches (approximately 17)	1	
The diameter of the large pipe measures $6 + 16.9 + 6 = 28.9$ inches (approx. **29** inches)	1	6
Total Points		**6**

Course Two – 2005

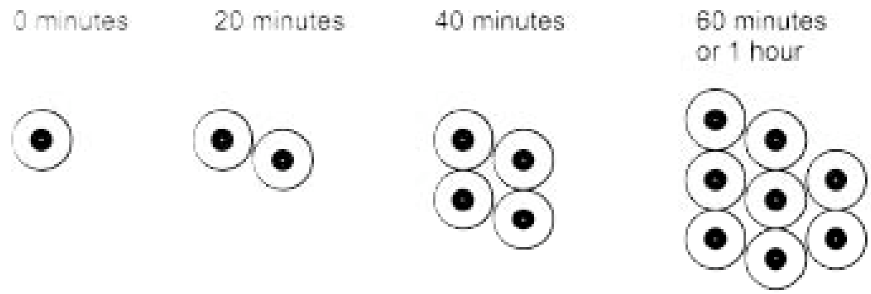
Course 2 Task 2 Multiplying Cells

Student Task	The task asks students to find and extend a number pattern based on powers of two and convert between pattern and time periods.
Core Idea 1 Functions	• **Understand patterns, functions, and relations.** • Understand and compare the properties of functions, including linear, quadratic, reciprocal and exponential functions.

Course Two – 2005

Multiplying Cells

This problem gives you the chance to:
- work with a sequence of numbers
- use powers of two



Mrs. Lucas's class has a 2-hour science lab.
She gives each student a dish with one cell in it.
She tells the class that in 20 minutes the cell will divide into two cells,
and each 20 minutes after that **each** cell in the dish will divide into two cells.

1. Complete the second row in this table to show how the number of cells increases during the lab.

Time (minutes)	0	20	40	60	80	100	120
Number of cells	1	2	4				
Number of cells as a power of 2	2^0	2^1					

2. Olan says that the numbers of cells can be written in the form 2^n.
   Complete the third row in the table to show how the number of cells can be written in this form.

3. Linda says that the number of cells after 3 hours will be 2^7 (= 2x2x2x2x2x2x2)

   Is she correct? ____________________

   If not, then what is the correct number?

   Explain how you figured it out.

   ____

   ____

   ____

4. How many cells will be in the dish after 5 hours?
   Give your answer as a normal number, not as a power of 2. ________________________

   Show how you figured it out.

5. How long will it take for the number of cells to reach at least 100,000?
   Give your answer to the nearest 20 minutes. ____________________________

   Show how you figured it out.

10

Multiplying Cells		Rubric
The core elements of performance required by this task are: • work with a sequence of numbers • use powers of two Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Fills in the table correctly: Time (minutes): 0, 20, 40, 60, 80, 100, 120 Number of cells: 1, 2, 4, **8**, **16**, **32**, **64** 4 correct values *Partial credit* 3 correct values	2 (1)	2
2. Fills in the table correctly: Time (minutes): 0, 20, 40, 60, 80, 100, 120 Number of cells as a power of 2: 2^0, 2^1, $\mathbf{2^2}$, $\mathbf{2^3}$, $\mathbf{2^4}$, $\mathbf{2^5}$, $\mathbf{2^6}$ 5 correct values *Partial credit* 4 correct values	2 (1)	2
3. Gives correct answer: $\mathbf{2^9}$ (allow 512) Gives a correct explanation such as: 3 hours is 9 lots of 20 minutes, and the power of 2 equals the number of 20 minutes which have passed.	1 1	2
4. Gives correct answer: 32,768 (accept 31.000 – 33,000) Shows correct work such as: 5 hours = 5 x 3 (lots of 20 minutes) = 15 lots of 20 minutes $\mathbf{2^{15}}$ = 32768	1 1ft	2
5. Gives correct answer: 340 minutes or 5 hours 40 minutes Shows correct work such as: 2^{16} = 32768 x 2 = 65536 2^{17} = 65536 x 2 = 131072 17 x 20 = 340	1 1ft	2
Total Points		**10**

Course Two – 2005

Course 2 Task 3 Sum of Two Squares

Student Task	The task asks students to find and use a pattern, including making an algebraic transformation to prove why a statement is true.
Core Idea 2 Mathematical Reasoning and Proofs	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples.** • Identify, formulate and confirm conjectures.
Core Idea 3 Algebraic Properties and Proofs	**Represent and analyze mathematical situations and structures using algebraic symbols.**
Core Idea 1 Functions	**Understand patterns, relations, and functions.** • Understand and perform transformations on functions.

Course Two – 2005

Sum of Two Squares

This problem gives you the chance to:
- find and use a pattern
- make and use an algebraic transformation

Lewis Carroll, the author of *Alice in Wonderland*, was also a mathematician.
In his diary for 1890, he wrote the following statement:

$2(x^2 + y^2)$ is always the sum of two squares

where x and y are a pair of non-zero integers.

Sarah wanted to prove this statement.

She began by testing it numerically in this way.

> If $x = 2$ and $y = 3$
>
> then, $2(x^2 + y^2) = 2(4 + 9)$
> $= 2 \times 13$
> $= 26$
> $= 1^2 + 5^2$
>
> which is the sum of two squares

1. Check whether this statement is true using at least two different pairs of values for x and y.

2. What is the relationship between the final two squares and the x and y numbers you started with?

____

____

____

3. Write $2(x^2 + y^2)$ algebraically as the sum of two squares.

____

4. Use algebra to prove that Lewis Carroll's statement is always true.

10

Course Two – 2005

Sum of Two Squares		Rubric
The core elements of performance required by this task are: • find and use a pattern • make and use an algebraic transformation Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Shows work such as: if x = 2 and y = 4 $2(x^2 + y^2) = 2(4 + 16) = 40 = 2^2 + 6^2$ if x = 2 and y = 5 $2(x^2 + y^2) = 2(4 + 25) = 58 = 3^2 + 7^2$	2 2	4
2. Gives a correct relationship such as: The final two squares are the square of the difference between x and y plus the square of the sum of x and y.	2	2
3. Gives correct answer: $2(x^2 + y^2)$ **$= (x - y)^2 + (x + y)^2$**	2	2
4. Shows correct work such as: $2(x^2 + y^2) = 2x^2 + 2y^2$ $= x^2 - 2xy + y^2 + x^2 + 2xy + y^2$ $= (x - y)^2 + (x + y)^2$	2	2
Total Points		**10**

Course Two – 2005

Course 2 Task 4 Quadrilaterals

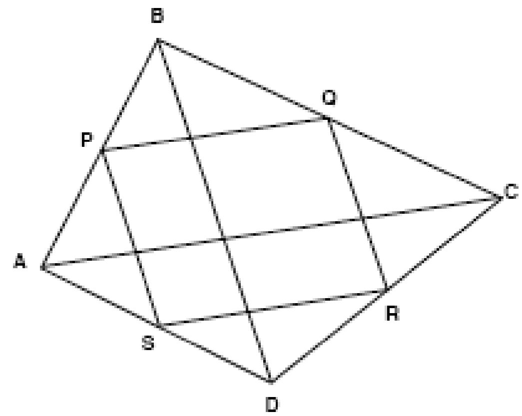
Student Task	The task asks students to use geometric properties to solve a problem about an inscribed quadrilateral.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two-dimensional geometric shapes; develop mathematical arguments about geometric relationships; and apply appropriate techniques, tools, and formulas to determine measurements.** • Explore relationships among classes of two-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.

Course Two – 2005

Quadrilaterals

This problem gives you the chance to:
- use geometric properties to solve a problem

ABCD is a quadrilateral. The points P, Q, R, S are the midpoints of the sides of the quadrilateral.



1. Write two correct statements about the lines PQ and AC.

2. What can you say about the quadrilateral PQRS? Explain your reasoning carefully.

3. If PQRS is a square, what can you say about the diagonals of ABCD?
   Explain how you figured it out.

8

Quadrilaterals Test 10

Course Two – 2005

Quadrilaterals		Rubric
The core elements of performance required by this task are: • use geometric properties to solve a problem Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. PQ and AC are parallel PQ is half the length of AC	1 1	2
2. Gives correct answer: PQRS is a **parallelogram** (with sides half the length of the diagonals of ABCD) Gives a correct explanation such as: Both PQ and SR are parallel to the diagonal AC, so they are parallel to each other. The lengths of both PQ and SR are half the length of the diagonal AC so they are equal in length. **or** Both PS and QR are parallel to the diagonal BD, so they are parallel to each other. The lengths of both PS and QR are half the length of the diagonal BD. A quadrilateral with two pairs of parallel sides **or** one pair of equal and parallel sides is a parallelogram.	1 2 **or** 2 1	4
3. Gives correct answer: If PQRS is a square, then the diagonals of ABCD are at right angles and are equal in length (because they are twice the length of the sides of PQRS). Accept two correct statements or One correct statement with explanation.	2	2
Total Points		**8**

Course Two – 2005

Course 2 Task 5 Ramps

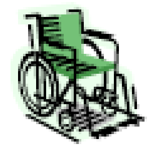
Student Task	The task asks students to use Pythagorean Theorem to find the radius of a large pipe with 4 smaller pipes fitting inside the larger pipe.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two-dimensional geometric shapes; develop mathematical arguments about geometric relationships; and apply appropriate techniques, tools, and formulas to determine measurements.**

Course Two – 2005

Ramps

This problem gives you the chance to:
- use trigonometry

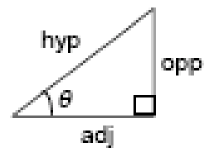
Simone is going to stay with the Gratto family.
Simone uses a wheelchair, so Mr. Gratto decides to make ramps so that it will be easier for her to get inside the house.



He thinks that the angle for the ramp should not be too great.

He decides that 15° is about right.

He measures the height of the front doorstep. It is 12cm high.



$\sin \theta = \frac{\text{opp}}{\text{hyp}}$

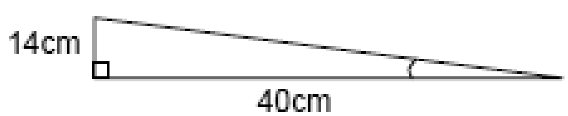
$\cos \theta = \frac{\text{adj}}{\text{hyp}}$

$\tan \theta = \frac{\text{opp}}{\text{adj}}$

1. How long will the sloping piece of wood for the ramp need to be?
   Show your calculations.

______________ cm

There is a limited space for a ramp outside the back door.
The step is 14cm high and there is 40cm for the base of the ramp.



2. What will be the measure of the angle for this ramp?
   Show how you figured it out.

______________ °

6

Ramps Test 10

Course Two – 2005

Ramps		Rubric
The core elements of performance required by this task are: • use trigonometry Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: 46 cm Shows correct work such as: $\sin 15^\circ = 12/x$ or $x = 12/\sin 15^\circ$	2 1	 3
2. Gives correct answer: 19° Shows correct work such as: $\tan x^\circ = 14/40$ or $x = \tan^{-1}(14/40)$ or $\arctan^{1}(14/40)$	2 1	 3
Total Points		**6**

Course Two – 2005

Geometry Task 1 London Eye

Student Task	Find the lengths and speeds on a rotating wheel. Reason about the relationship between function of speed and its graph over time.
Geometry & Measurement	**Analyze characteristics and properties of two- and three-dimensional geometric shapes; develop mathematical arguments about geometric relationships; and apply appropriate techniques, tool, and formulas to determine measurements.**
Algebraic Properties and Representations	Recognize and use equivalent graphical and algebraic representations.

Geometry – 2006

London Eye

This problem gives you the chance to:
• find lengths and speeds on a rotating wheel

The London Eye, on the south bank of the Thames River, was completed in the year 2000 as a project for the new millennium.

It is the world's largest Ferris wheel, with a diameter of 135 meters.

A ride on the London Eye, one complete revolution, takes 30 minutes.

Passengers step on at the bottom of the wheel as it slowly turns at a constant speed.

The circumference of a circle is equal to its diameter multiplied by π.

1. What is the circumference of the London Eye wheel? ____________ meters

   Show your calculations.

2. At what speed, in meters per minute, are passengers on the wheel travelling?

   ____________ meters per minute

   Show how you figured it out.

3. Passengers start the ride at the lowest part of the wheel.
   How high above the starting point are they after one quarter of the ride? __________ meters

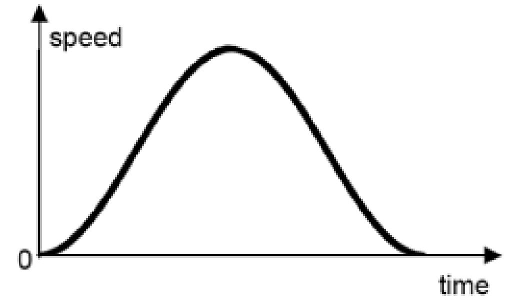
   Explain your reasoning.

   ____

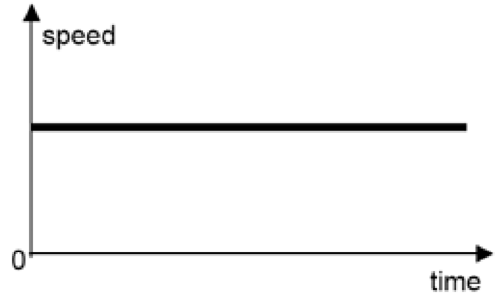
   ____

Geometry – 2006

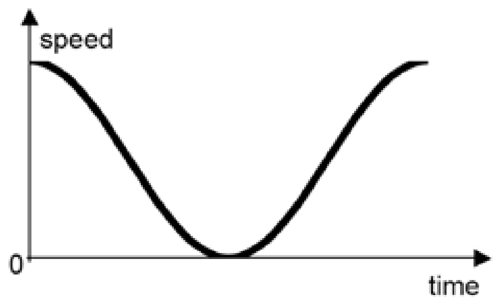
A



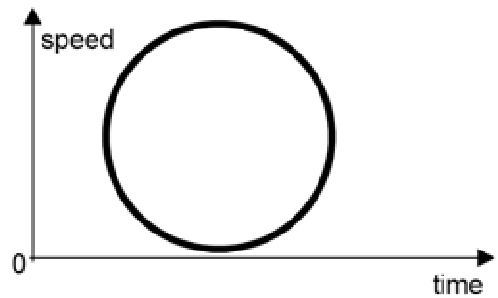
B



C



D



4. Which of the four graphs, A, B, C or D, shows the speed of passengers on the London Eye during the ride?

Graph _______________

Explain how you figured it out.

____

____

____

7

London Eye Rubric

The core elements of performance required by this task are: • find lengths and speeds on a rotating wheel Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **424** meters (to the nearest meter) accept 135π	1	
Shows work such as: $\pi \times 135$	1	2
2. Gives correct answer: **14** meters per minute(to the nearest meter) accept 4.5π	1	
Shows correct work such as: speed $= \frac{\text{distance}}{\text{time}} = \frac{424}{30}$ meters per minute or $\frac{135\pi}{30}$	1	2
3. Gives correct answer: **67.5** meters **and** Gives a correct explanation such as: This is half of the diameter.	1	1
4. Gives correct answer: **B**	1	
Gives a correct explanation such as: The speed is constant, so the graph is a horizontal line (dependent on B).	1	2
Total Points		**7**

Geometry – 2006

Geometry Task 2 Hopewell Geometry

Student Task	Work with the Pythagorean Rule, angles and similarity in given triangles. Write a justification for whether or not a triangle in a diagram has a right angle.
Core Idea 2 Mathematical Reasoning and Proof	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem, including deductive and inductive reasoning, making and testing conjectures and using counter examples and indirect proof.**
Core Idea 3 Geometry & Measurement	**Analyze characteristics and properties of two- and three-dimensional geometric shapes; develop mathematical arguments about geometric relationships; and apply appropriate techniques, tool, and formulas to determine measurements.**

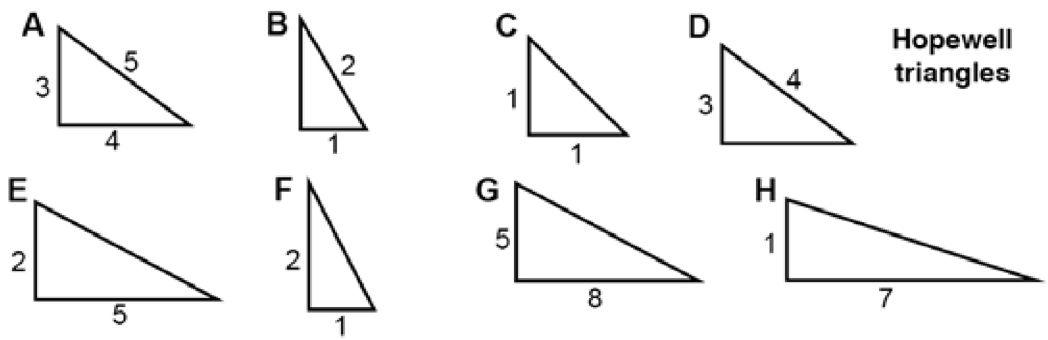
Geometry – 2006

Hopewell Geometry

This problem gives you the chance to:
- work with the Pythagorean Rule, angles and similarity in given triangles

The Hopewell people were Native Americans whose culture flourished in the central Ohio Valley about 2000 years ago.

The Hopewell people constructed earthworks using right triangles, including those below.

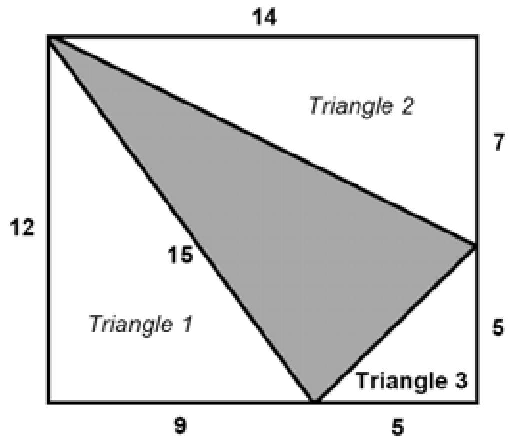


1. What is the length of the hypotenuse of Triangle H?
   Give your answer correct to one decimal place. _______________
   Show your calculation.

2. What is the size of the smallest angle in Triangle A?
   Give your answer correct to one decimal place. _______________
   Show your calculation.

The diagram on the next page shows the layout of some Hopewell earthworks. The centers of the Newark Octagon, the Newark Square and the Great Circle were at the corners of the shaded triangle.

Geometry – 2006



Not drawn to scale

The three right triangles surrounding the shaded triangle form a rectangle measuring 12 units by 14 units.

Each of these three right triangles is similar to one of the Hopewell triangles on the previous page.

For example, Triangle 3 above is similar to Hopewell Triangle C.

3. Which Hopewell triangle is similar to Triangle 1? ______________

Explain how you decided.

____

____

____

4. Is the shaded triangle a right triangle? ______________

Explain how you decided, showing all your work.

____

____

____

8

Hopewell Geometry — Rubric

The core elements of performance required by this task are: • work with the Pythagorean Rule, angles and similarity in given triangles Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **7.1** (accept 7 or $5\sqrt{2}$) Shows correct work such as: $\sqrt{(1^2 + 7^2)}$	1 1	2
2. Gives correct answer: **36.8° to 36.9°** Shows correct work such as: $\sin^{-1}\frac{3}{5}$ or $\cos^{-1}\frac{4}{5}$ or $\tan^{-1}\frac{3}{4}$	1 1	2
3. Gives correct answer: Triangle **A** Gives correct explanation such as: Triangle 1 is an enlargement of Triangle A by a scale factor of 3.	1 1	2
4. Gives correct answer: **No,** **and** Gives a correct explanation such as by finding lengths of all three sides ($\sqrt{225}$, $\sqrt{50}$, $\sqrt{245}$) and showing they don't satisfy the Pythagorean Rule. $245 \neq 225 + 50$. *Other methods include:* • Using trigonometry to find the angles (71,6, 81.9, 25.5) • Triangle 3 is isosceles ∴ it has two 45° angles. Triangles 1 and 2 are not isosceles ∴ do not have 45° angles. Angle in shaded triangle = 180° - 45° - non 45° angle ∴ ≠ 90° *Partial credit* Gives a partially correct explanation.	2 (1)	2
Total Points		**8**

Geometry – 2006

Geometry Task 3 Quadratic

Student Task	Find graphical properties of a quadratic function given its formula.
Core Idea 1 Functions	**Understand patterns, relations, and functions.** • Understand and perform transformations on functions. • Understand properties of functions including quadratic functions.
Core Idea 2 Mathematical reasoning and proofs	Identify, formulate and confirm conjectures.

Geometry – 2006

Quadratic

This problem gives you the chance to:
• find graphical properties of a quadratic function given by its formula

$y = x^2 - 3x - 10$ is a quadratic function.

Say whether each of these statements about the function is true or false.

If a statement is false, give a true version of the statement.

1. The graph of $y = x^2 - 3x - 10$ cuts the y-axis at (0, -10). ____________________

2. $y = x^2 - 3x - 10$ can be written as $y = (x-2)(x+5)$. ____________________

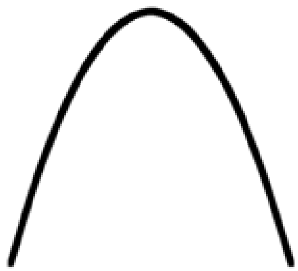
3. When $x =$ -3, $y =$ - 10. ____________________

4. The solutions of the equation $x^2 - 3x - 10 = 0$ are x = 2 and x = 5. ____________________

5. The function has a minimum value but no maximum value. ____________________

6. The graph of $y = x^2 - 3x - 10$ is below the x-axis for $-2 \le x \le 5$. ____________________

7. The graph of $y = x^2 - 3x - 10$ looks like this: ____________________



8. The graph of $y = x^2$ can be transformed into the graph of $y = x^2 - 3x - 10$ by translations and/or stretches.

____

9. Identify the transformations required to transform $y = x^2 - 3x - 10$ into $y = x^2$.

9

Quadratic		Rubric
The core elements of performance required by this task are: • find graphical properties of a quadratic function given by its formula Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **true**	1	1
2. Gives correct answer: **false, y = (x + 2)(x – 5)**	1	1
3. Gives correct answer: **false, y = 8**	1	1
4. Gives correct answer: **false, x = -2 and 5**	1	1
5. Gives correct answer: **true**	1	1
6. Gives correct answer: **false and –2 < x < 5** Accept **true** (B.O.D)	1	1
7. Gives correct answer: **false, should be other way up**	1	1
8. Gives correct answer: **true**	1	1
9. Gives correct answer such as: A translation of 3/2 in the *x* direction right and –49/4 in the *y* direction down, or the reverse: 3/2 up and 49/4 down.	1	1
Total Points		**9**

Geometry – 2006

Geometry **Task 4** **How Compact?**

Student task	Use a formula for a new concept. Calculate areas. Use the Pythagorean Theorem.
Core Idea 3 Algebraic Properties & Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Develop fluency in operations with real numbers. • Solve equations involving radicals and exponents in contextualized problems such as use of Pythagorean Theorem.
Core Idea 4 Geometry & Measurement	• Understand and use formulas for the area, surface area, and volume of geometric figures.

Geometry – 2006

How Compact?

This problem gives you the chance to:
- use a formula for a new concept
- calculate areas
- use the Pythagorean Rule

The compactness of something tells us how closely packed together it is.
The compactness of a shape can be calculated using this formula:

$$C = \frac{4A}{\pi d^2}$$

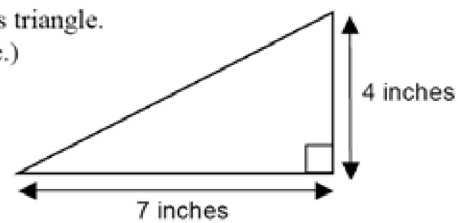
where **A** is the **area** of the shape and **d** is the **distance** between the two points of the shape that are **furthest apart**.

Using this formula, C always lies between 0 and 1. A perfectly compact shape has a measure of 1.

1. Use the formula to calculate the compactness of this triangle.
   (The distance d will be the length of the hypotenuse.)

   ____

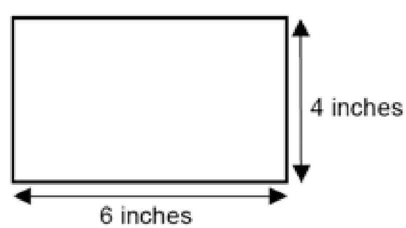
   Show your calculation.



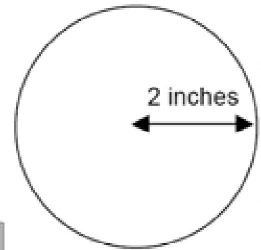
2. Calculate the compactness of this rectangle.

   ____

   Show your calculation.



3. Calculate the compactness of a circle with radius 2 inches.



____

The area of a circle is equal to π multiplied by the square of its radius

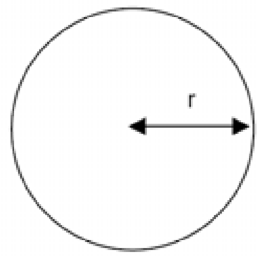
Explain how you figured it out.

____

____

____

4. Calculate the compactness of a circle of radius r.



____

5. What does your answer to question 4 tell you about all circles?

____

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8

How Compact? Rubric

The core elements of performance required by this task are: • use a formula for a new concept • calculate areas • use the Pythagorean Rule Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **0.27** accept 0.27 – 0.3 or $\frac{56}{65\pi}$	1	
Shows correct work such as: $d^2 = 7^2 + 4^2 = 65$ or $d = 8.1$ $(4 \times 4 \times 7 \div 2) \div (\pi \times 65)$	1	2
2. Gives correct answer: 0.58 accept 0.58 – 0.6 or $\frac{96}{52\pi}$	1	
Shows correct work such as: $d^2 = 6^2 + 4^2 = 52$ or $d = 7.2$ $(4 \times 4 \times 6) \div (\pi \times 52)$	1	2
3. Gives correct answer: **1** accept 0.9999……	1	
Shows correct work such as: $A = \pi \times 2^2$ (= 12.566) $(4 \times \pi \times 2^2) \div (\pi \times 4^2)$	1	2
4. Gives correct answer: **1**	1	1
5. Gives correct explanation such as: All circles are perfectly compact.	1	1
Total Points		**8**

Geometry – 2006

Geometry **Task 5** **Rhombuses**

Student task	Use the properties of a rhombus. Use Pythagorean Theorem. Make a justification for similarity.	
Core Idea 2 Mathematical Reasoning & proof	Employ forms of mathematical reason and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples.	
	• Explain the logic inherent in a solution process.	
Core Idea 3 Geometry & Measurement	Analyze characteristics and properties of two and three-dimensional geometric shapes; develop mathematical arguments about geometric relationship; and apply appropriate techniques, tool, and formulas to determine measurements.	
	• Understand and use formulas, including solving Pythagorean theorem and trig functions. • Make and test conjectures about geometric objects.	

Geometry – 2006

Rhombuses

This problem gives you the chance to:
- use properties of a rhombus
- use the Pythagorean Rule
- check figures for similarity

1. Here is a rhombus PQRS drawn in an 8cm by 10cm rectangle ABCD.
   P, Q, R and S are the mid-points of the sides of the rectangle.

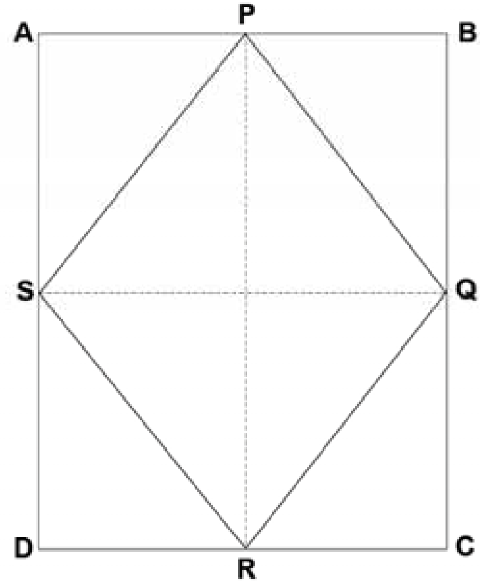
   Explain clearly why PQRS is a rhombus.

   ____

   ____

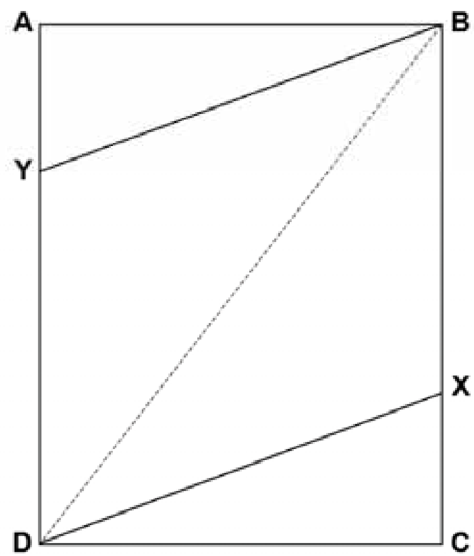
   ____

   ____



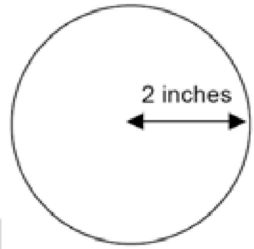
2. Here is a parallelogram BXDY drawn in the 8cm by 10cm rectangle ABCD.

   Show that BXDY is a rhombus if AY and CX are 1.8cm.



Geometry – 2006

3. Calculate the compactness of a circle with radius 2 inches.



____

The area of a circle is equal to π multiplied by the square of its radius

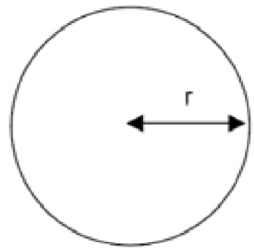
Explain how you figured it out.

____

____

____

4. Calculate the compactness of a circle of radius r.



____

5. What does your answer to question 4 tell you about all circles?

____

____

____

____

8

Rhombuses — Rubric

The core elements of performance required by this task are: • use properties of a rhombus • use the Pythagorean Rule • check figures for similarity Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives a correct explanation such as: PQRS is a quadrilateral with four sides of equal measure with explanation. *Partial credit* PQRS is a quadrilateral with four sides of equal measure	 2 (1)	 2
2. Shows that BY = XD = **8.2** cm With correct work such as: $BY^2 = AY^2 + AB^2$ $BY^2 = 1.8^2 + 8^2$ BY = 8.2	1 1	 2
3. In rhombus PQRS, shows that PS = 6.4 In rhombus BXDY, shows that BD = 12.8 Shows that the sides of the triangles PSR and BYD are proportional. Shows that: $\frac{PS}{BY} = \frac{PR}{BD}$ e.g. accept $\frac{6.4}{8.2} = 0.78$ and $\frac{10}{12.8} = 0.78$ Since the sides are proportional, the rhombuses are similar. *Alternatively* Finds angle SPQ = angle YBX = 77.3° Or angle PSR = angle BYD = 102.7° Since the angles are equal, the rhombuses are similar.	1 1 1 1 **or** 2 2	 4
Total Points		**8**

Geometry – 2006

Geometry Task 1 Insane Propane

Student Task	Work with volumes of cylinders and spheres. Reason about and quantify how change in one dimension effects volume.
Core Idea 4 Geometry and Measurement	**Apply appropriate techniques, tools, and formulas to determine measurements.** • Understand and use formulas for the area, surface area, and volume of geometric figures, including spheres and cylinders.

Mathematics of the task:
- Ability to compose and decompose a shape to find volume
- Compute accurately with exponents
- Understand calculations with algebraic expressions by knowing when and how to combine like terms, how to use substitution, and to use variables to write generalizations
- Understand how a change in one dimension effects shape and volume of an object and be able to describe and quantify the relationships

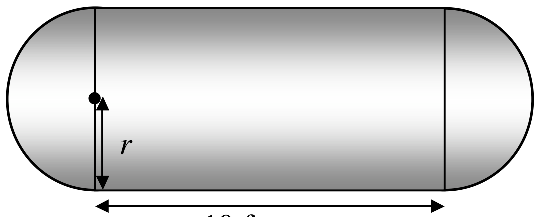
Based on teacher observations, this is what geometry students knew and were able to do:
- Use a formula to find the volume of a sphere
- Make a mathematical argument about why doubling the radius does not double the volume

Areas of difficulty for geometry students:
- Finding both the volume of a cylinder and the total volume of the tank
- Computing with exponents
- Writing a formula for the volume of a complex object
- Confusing the volume for part of an object for the volume of the whole object

Insane Propane

This problem gives you the chance to:
• work with volumes of cylinders and spheres

People who live in isolated or rural areas have their own tanks of gas to run appliances like stoves, washers, and water heaters. These tanks are made in the shape of a cylinder with hemispheres on the ends.



The Insane Propane Tank Company makes tanks with this shape, in different sizes. The cylinder part of every tank is exactly 10 feet long, but the radius of the hemispheres, r ,will be different depending on the size of the tank.

(The volume of a cylinder is given by the formula $V = \pi r^2 h$ *and the volume of a sphere is given by the formula* $V = \frac{4}{3}\pi r^3$*.)*

1. Tank A has a radius of 3 feet. What is its volume? ____________ cubic feet

Show how you figured it out.

2. Tank B has a radius twice as big as tank A.
   Is the volume of tank B twice as big as the volume of tank A? ________________

Explain your answer clearly.

____

____

3. Check your answer to question # 2 by figuring out the volume of a tank with a 6-foot radius.

Show how you figured it out below. ______________cubic feet

4. Write a formula that will let Insane Propane determine the volume of a tank with any size radius.

____

8

Task 1: Insane Propane		Rubric
The core elements of performance required by this task are: • work with volumes of cylinders and spheres Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **396 or 126π** accept 395-396 cubic feet	1	
Shows work such as:		
Cylinder $\pi r^2 h = 10\pi 3^2 = 283$ or 90π	1	
Sphere $\frac{4\pi r^3}{3} = \frac{4\pi 3^3}{3} = 113$ or 36π	1	3
2. Gives correct answer: **No** and Gives correct explanation such as: Although the radius is twice as big, we need to square it when calculating the volume of the cylinder and cube it when calculating the volume of the hemispheres. This will make the volume much more than twice as big.	1	1
3. Gives correct answer: **2036 or 648π** accept 2034 – 2036 cubic feet	1	
Shows correct work such as: Cylinder = $10\pi 6^2 = 1131$ or 360π	1	
Sphere = $\frac{4\pi 6^3}{3} = 905$ or 288π	1	3
4. Gives correct answer such as: Volume = $10\pi r^2 + \frac{4}{3}\pi r^3$	1	
Accept $h\pi r^2 + \frac{4}{3}\pi r^3$		1
Total Points		**8**

Geometry Task 2 Sloping Squares

Student Task	Calculate areas in composite figures formed by squares and rectangles. Use and manipulate algebraic expressions in this situation.
Core Idea 3 Algebraic Properties and Representations.	**Represent and analyze mathematical situations and structures using algebraic symbols.**
Core Idea 4 Geometry and Measurement	**Apply appropriate techniques, tools, and formulas to determine measurements.**

Mathematics in this task:

- Compose and decompose shapes
- Use formulas to find area of squares and triangles, both with values and algebraically
- Multiply binomials
- Simplify algebraic expressions
- Use substitution and solve an expression

Based on teacher observations, this is what geometry students knew and were able to do:

- Find the numeric values for area of a square and a triangle
- Use a diagram to explain in algebraic terms the length of a side in terms of other known quantities

Areas of difficulty for geometry students:

- Relating algebraic expressions to a context
- Using algebra to prove geometric relationships
- Solving algebraic expressions

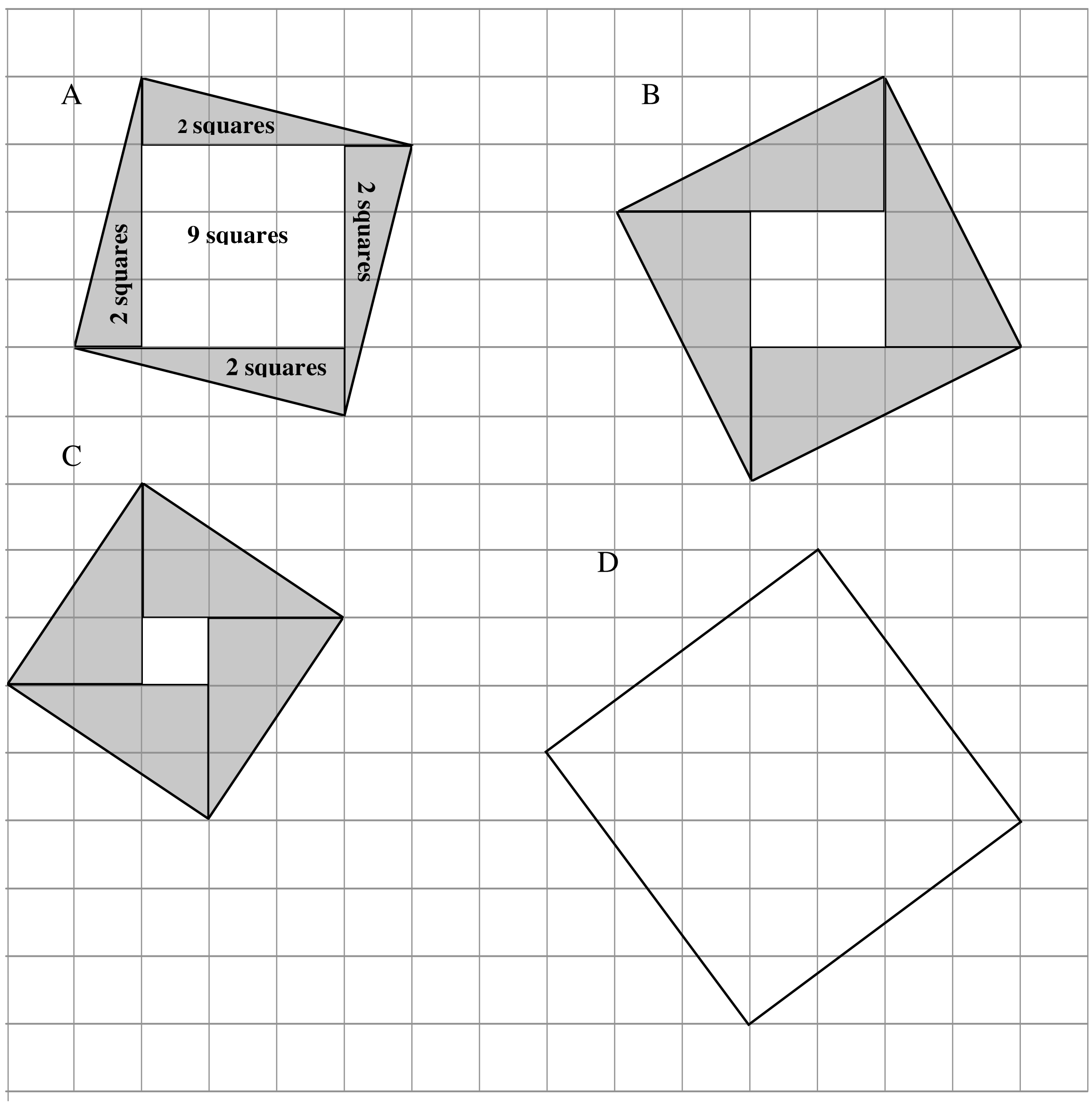
Sloping Squares

This problem gives you the chance to:

- calculate areas in composite figures formed of squares and rectangles
- use and manipulate algebraic expressions in this situation

The diagram shows some 'sloping squares' drawn on a grid.

Sloping Square A is split into four equal right triangles and a central square.
The square in the center has area 9 squares and each of the four triangles has area 2 squares. This shows that the total area is equal to 17 grid squares.

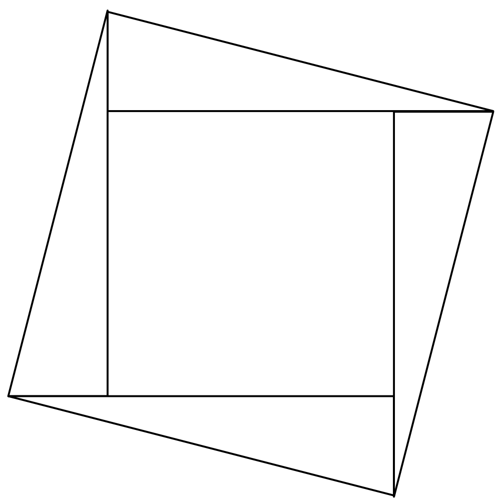


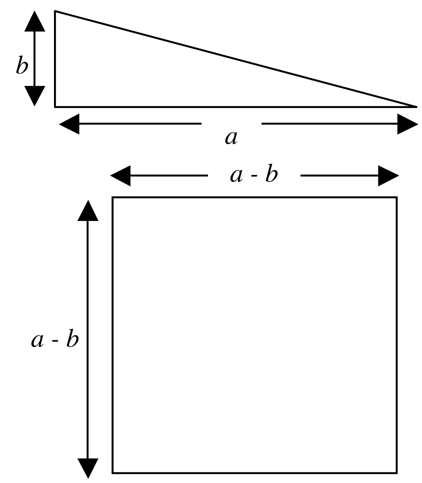
1. a. Show that Sloping Square B has a total area of 20 grid squares.

b. Find the area of Sloping Square C. _______________

c. Find the area of Sloping Square D. _______________

2. The diagram below shows a sloping square split into four 'a by b' right triangles and a central square.





a. Show that the total area of the four right triangles is $2ab$.

b. Use the diagram to show why the sides of the square are $a - b$ units long.

c. Find the area of the central square. Write your expression without parentheses.

d. Show that the total area of the sloping square (four triangles and the central square) is $a^2 + b^2$.

e. Show that this formula for the total area of a sloping square gives the correct answer of 17 grid squares for Sloping Square A.

Sloping Squares		Rubric
The core elements of performance required by this task are: • calculate areas in composite figures formed of squares and rectangle • use and manipulate algebraic expressions in this situation Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1.a Shows correct work such as: (Possibly shown on diagram) Four triangles of 4 squares each Central square 4 squares Total 20 squares	1	
b Gives correct answer: **13** squares	1	
c Gives correct answer: **25** squares	1	3
2.a Shows correct work such as: $\frac{1}{2}ab \times 4 = 2ab$	1	
b Correct indication on diagram or other explanation	1	
c Shows correct work such as: $(a-b)^2 = a^2 - 2ab + b^2$	2	
d Shows correct work such as: $a^2 - 2ab + b^2 + 2ab = a^2 + b^2$	1	
e Shows correct work such as: $a = 4,\ b = 1$ $a^2 + b^2 = 16 + 1 = 17$	1	6
Total Points		**9**

Geometry Task 3 Sum and Product

Student Task	Use arithmetic and algebra to represent and analyze a mathematical situation and solve a quadratic equation by trial and improvement.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Solve equations involving radicals and exponents in contextualized problems.

Mathematics of the task:

- Ability to read and interpret constraints and make sense of a table of values
- Ability to make mathematical descriptions of information in a table and quantify relationships
- Understand and use guess and check and/or use the quadratic formula to find an optimal solution to match the constraints of the task
- Recognize multiple representations of expressions for two sets of constraints to describe the relationship between two variables

Based on teacher observations, this is what geometry students knew and were able to do:

- Solving for *b* and finding the product of *a* and *b* in part 3b of the task.
- Describing how to find *b* in part 1a of the task
- Describing why *a* must be located in a particular place on the number line based on values in the table in 3a

Areas of difficulty for geometry students:

- Calculating and comparing with decimals
- Identifying algebraic expressions to show the relationship between *a* and *b* if their product is 10
- Describing the trends in the product column of the table
- Explaining why a must be between 1 and 1.5
- Finding the values for *a* and *b* to two decimal points
- Solving a quadratic expression that doesn't factor

Sum and Product

This problem gives you the chance to:

- use arithmetic and algebra to represent and analyze a mathematical situation
- solve a quadratic equation by trial and improvement

Tom wants to find two real numbers, a and b, that have a sum of 10 and have a product of 10.
He makes this table.

Sum of *a* and *b*	***a***	***b***	**Product of *a* and *b***
10	0	10	0
10	1	9	9
10	2	8	16
10	3	7	21
10	4	6	24
10	5	5	25
10	6	4	24
10	7	3	21
10	8	2	16
10	9	1	9
10	10	0	0

1\. a. How can Tom find b if he knows a? ____

____

b. Draw a ring around any of these algebraic statements that express the relationships between a and b correctly.

$a + b = 10$ $\quad$ $b = a + 10$ $\quad$ $b = 10 - a$ $\quad$ $a = 10 - b$

$a = 10b$ $\quad$ $b = 10a$ $\quad$ $ab = 10$ $\quad$ $b = \dfrac{10}{a}$

2\. Explain what happens to the product of a and b (in the last column of the table) as a increases from 0 to10.

____

____

3. Tom looks at the table and says, “If the product of *a* and *b* is 10, *a* must be somewhere between 1 and 2 or between 8 and 9.”

a. Explain how the table shows this. ____

____

____

b. Tom tries to find the value between 1 and 2. He decides to try $a = 1.5$ Complete this table to show his calculations.

Sum of *a* and *b*	***a***	***b***	**Product of *a* and *b***
10	1.5		

c. Which of these two statements is correct? Explain how you decided.

- *a* must be between 1 and 1.5
- *a* must be between 1.5 and 2

____

____

____

d. Find the values of *a* and *b* correct to two decimal places. ____________________

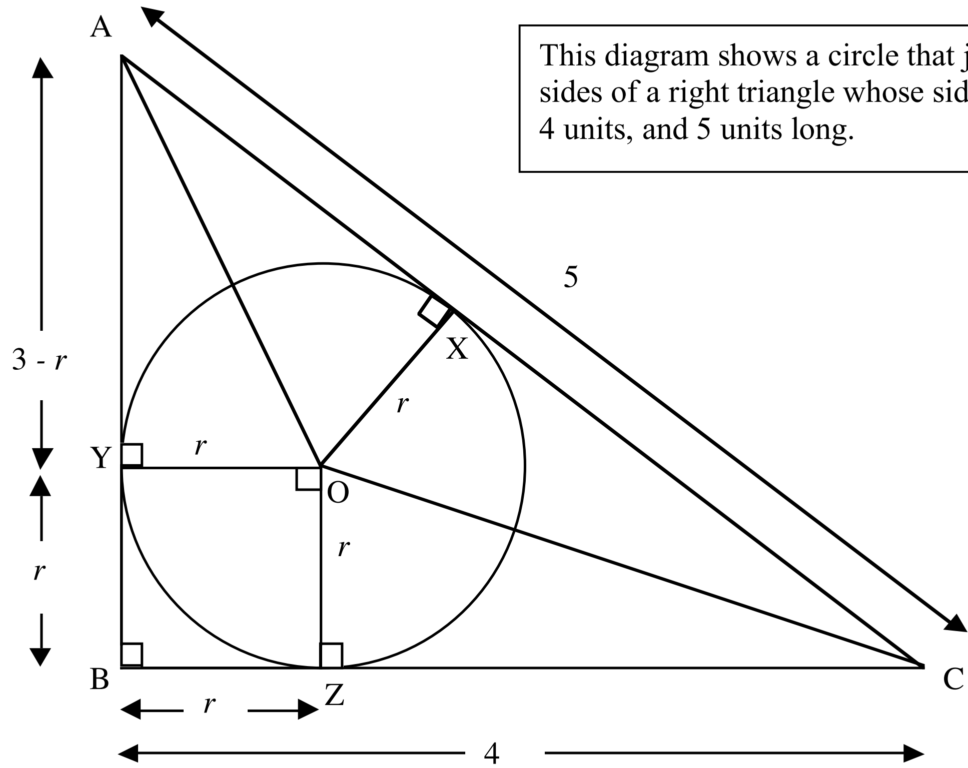
Show your calculations.

8

Sum and Product		Rubric
The core elements of performance required by this task are: • use arithmetic and algebra to represent and analyze a mathematical situation • solve a quadratic equation by trial and improvement Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. a. Gives correct answer: Subtract *a* from 10 or divide 10 by *a*.	1	
b. Draws rings around $a+b=10$ $\quad b=a+10$ $\quad b=10-a$ $\quad a=10-b$ $\quad a=10b$ $\quad b=10a$ $\quad ab=10$ $\quad b=\frac{10}{a}$	1	2
2. Gives correct explanation such as: The product starts at zero, increases up to a maximum of 25, then decreases back to zero.	1	1
3.a Gives correct explanation such as: The product is less than 10 when *a* is 1 and more than 10 when *a* is 2, so *a* must be between 1 and 2. Similarly for 8 and 9.	1	
b Gives correct answer: **8.5, 12.75**	1	
c Gives correct explanation such as: The product is less than 10 when *a* is 1 and more than 10 when *a* is 1.5, so *a* must be between 1 and 1.5	1	
d Gives correct answers: **1.13** and **8.87**	1	
Shows correct work such as: 1.13 x 8.87 = 10.0231 1.12 x 8.88 = 9.9456	1	5
Total Points		**8**

Circles in Triangles

This problem gives you the chance to:
• use algebra to explore a geometric situation



This diagram shows a circle that just touches the sides of a right triangle whose sides are 3 units, 4 units, and 5 units long.

1. Explain why triangles AOX and AOY are congruent.

____

____

____

2. What can you say about the measures of the line segments CX and CZ?

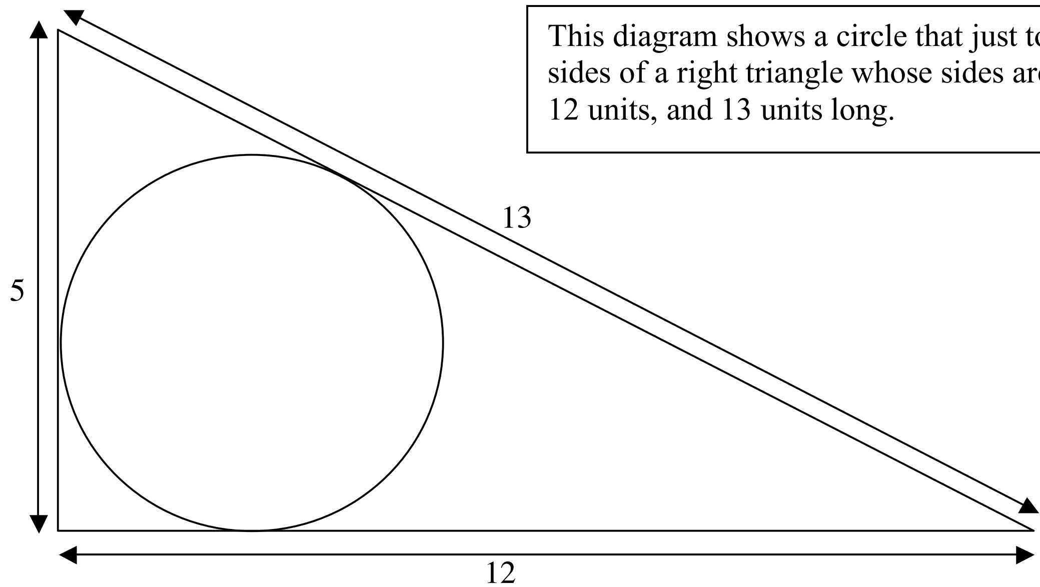
____

____

____

3. Find r, the radius of the circle. Explain your work clearly and show all your calculations.

4.



This diagram shows a circle that just touches the sides of a right triangle whose sides are 5 units, 12 units, and 13 units long.

Draw construction lines as in the previous task, and find the radius of the circle in this 5, 12, 13 right triangle. Explain your work and show your calculations.

9

Circles in Triangles — Rubric

The core elements of performance required by this task are: • use algebra to explore a geometric situation Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Triangle AOY is congruent to triangle AOX (SAS) or (SSS) or (Hypotenuse, leg)	2	
Partial credit For a partially correct explanation	(1)	2
2. States that: CZ = CX	1	1
3. Gives correct answer **r = 1**	1	
Shows correct work such as: CX = 4 – r = 3 = CZ CX = 5 - AX $5 = 3 - r + 4 - r$ or $r^2 + r(3- r) + r(4 - r) = 6$	2	
Partial credit Shows partially correct work or uses guess and check. Alternatively, may use area reasoning	(1)	3
4. Draws in construction lines and uses a similar method to Question #3,	1	
Gives correct answer **r = 2**	1	
Shows correct work such as: $13 = 5 - r + 12 - r$ or $r^2 + r(5- r) + r(12 - r) = 30$ Alternatively, may use area arguments.	1	3
Total Points		**9**

Algebra Task 5 Two Solutions

Student Task	Find solutions to equations and inequalities.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Write equivalent forms of equations, inequalities and systems of equations and solve them. • Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations.

Mathematics in this task:

- Ability to understand what is meant by the solution to an equation or inequality.
- Ability to calculate solutions to inequalities and equalities.
- Ability to use exponents, negative numbers and square roots.
- Ability to think about classes of numbers and the difference between finite and infinite sets
- Ability to recognize properties of equations and inequalities with regards to the number of possible solutions
- Ability to understand variable in a variety of ways

Based on teacher observations, this is what algebra students knew and were able to do:

- Find two solutions for $1776x + 1066 \geq 365$
- Find both solutions for $x^2=121$
- Find $x^2 > x^3$ and for $|x| > x$

Areas of difficulty for algebra students:

- Understanding variable
- Understanding infinity
- Finding solutions for inequalities
- Identifying equations that have a limited number of solutions
- Thinking about classes of equations
- Working with exponents
- Understanding that the x's represent the same number with an equation

Two Solutions

This problem gives you the chance to:
- find solutions to equations and inequalities

1. For each of the following equalities and inequalities, find two values for x that make the statement true.

a. $x^2 = 121$ __________ __________

b. $x^2 = x$ __________ __________

c. $x^2 < x$ __________ __________

d. $(x-1)(5x^4 - 7x^3 + x) = 0$ __________ __________

e. $1776x + 1066 \geq 365$ __________ __________

f. $x^2 > x^3$ __________ __________

g. $|x| > x$ __________ __________

2. Some of the equations and inequalities on the page opposite have exactly two solutions; others have more than two solutions.

a. Write down two equations or inequalities that have exactly two solutions. Explain your answer.

b. Write down one equation or inequality that has more than two solutions, but not infinitely many solutions. Explain your answer.

c. Write down two equations or inequalities that have an infinite number of solutions.

10

Task 5: Two Solutions — Rubric

The core elements of performance required by this task are: • find solutions to equations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: a: **± 11** b: **0, 1** c: any values between **0 and 1** d: **0, 1** e: any value $\geq$ **-0.3947** f: any value less than **1 except 0** g: any negative value	7 x 1	7
2. Gives correct answers with reasons such as: a. $\mathbf{x^2 = 121}$ and $\mathbf{x^2 = x}$ These are quadratic equations with two roots b. $\mathbf{(x - 1)(5x^4 - 7x^3 + x) = 0}$ c. Gives two of: $\mathbf{x^2 < x,\ 1776x + 1066 \geq 365,\ x^2 > x^3,}\ \|x\| > x$	1 1 1	3
Total Points		**10**

Geometry Task 1 Numbers

Student Task	Prove number relations using algebra.
Core Idea 1 Functions	**Understand patterns, relations, and functions.** • Understand and perform transformations on functions.
Core Idea 2 Mathematical Reasoning and Proofs	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples and indirect proof.** • Show mathematical reasoning in solutions in a variety of ways including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models. • Explain the logic inherent in a solution process. • Identify, formulate and confirm conjectures.

The mathematics of this task:

- Find numerical examples to follow a rule
- Recognize a pattern of square numbers
- Write a generalization about a pattern
- Understand how to quantify an even number algebraically
- Looking for multiple relationships and being able to express them using the fewest number of variables

Based on teacher observations, this is what geometry students know and are able to do:

- Recognize square numbers
- Follow a numeric rule

Areas of difficulty for geometry students:

- Expressing an even number symbolically
- Looking at generalizable patterns rather than recursive patterns

Geometry – 2008

Numbers

This problem gives you the chance to:
• prove number relations using algebra

1. What kind of numbers do you get when you multiply two numbers that differ by 2, then add 1? Show your work.

2. What kind of numbers do you get when you multiply two numbers that differ by 4, then add 4? Show your work.

3. What kind of numbers do you get when you multiply two numbers that differ by 6, then add 9? Show your work.

4. Find a general rule. Explain your reasoning using algebra.

8

Geometry – 2008

Numbers		Rubric
The core elements of performance required by this task are: • prove number relations using algebra Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: square numbers Shows work such as: $1 \times 3 + 1 = 4$, $2 \times 4 + 1 = 9$	1 1	2
2. Gives correct answer: square numbers Shows work such as: $1 \times 5 + 4 = 9$, $2 \times 6 + 4 = 16$	1 1	2
3. Gives correct answer: square numbers Shows work such as: $1 \times 7 + 9 = 16$, $2 \times 8 + 9 = 25$	1 1	2
4. Gives correct answer such as: When you multiply two numbers that differ by 2n, then add n^2 you get square numbers. Shows correct work such as: $a \times (a + 2n) + n^2 = a^2 + 2an + n^2 = (a + n)^2$	1 1	2
Total Points		**8**

Geometry – 2008

Geometry Task 2 Glasses

Student Task	Calculate volumes of cylinders and cones in a practical context.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two- and three-dimensional shapes; develop mathematical arguments about geometric relationships; apply appropriate techniques, tools, and formulas to determine measurements.** • Understand and use formulas for area, surface area, and volume of geometric figures, including spheres and cylinders. • Visualize three-dimensional objects from different perspectives and analyze their cross sections.

The mathematics of this task:

- Constructing and deconstructing diagrams of 3-dimensional shapes
- Using Pythagorean theorem
- Using formulas to find volume of 3-dimensional shapes: cylinders, spheres, and cones
- Problem-solving around filling an irregularly shaped glass, from the bottom up
- Working backwards from the volume to the height of a cylinder

Based on teacher observations, this is what geometry students knew and were able to do:

- Use formulas
- Use Pythagorean theorem

Areas of difficulty for geometry students:

- Decomposing glass b into a cylinder and a half-sphere
- Thinking about filling a glass from the bottom up
- Working backwards from a volume to a height
- Interpreting legs and hypotenuse in a drawing
- Confusing half the height for half the volume

Geometry – 2008

Glasses

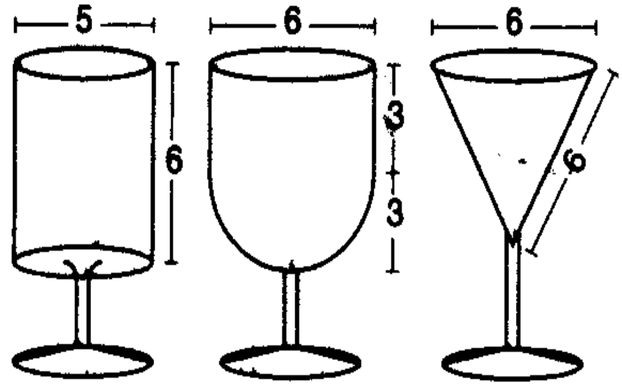
This problem gives you the chance to:
- calculate volumes of cylinders and cones in a practical context

This diagram shows three glasses (not drawn to scale).
The measurements are all in centimetres.

The volume of a cylinder $= \pi r^2 h$

The volume of a sphere $= \dfrac{4\pi r^3}{3}$

The volume of a cone $= \dfrac{\pi r^2 h}{3}$



1 2 3

The bowl of glass 1 is cylindrical. The diameter is 5 cm and the height is 6 cm.

The bowl of glass 2 is a cylinder with a hemispherical bottom. The diameter is 6 cm and the height of the cylinder is 3 cm.

The bowl of glass 3 is an inverted cone. The diameter is 6 cm and the slant height is 6 cm.

1. Find the vertical height of the bowl of glass 3. ___________ cm.
   Show your work.

2. Calculate the volume of the bowl of each of these glasses. Show your work.

a. Glass 1 ____________ cm^3

b. Glass 2 ____________ cm^3

Geometry – 2008

c. Glass 3 ____________ cm^3

3. Find the height of liquid in Glass 2 when it is half full. ____________ cm
Show your calculations

8

Geometry – 2008

Glasses		Rubric
The core elements of performance required by this task are: • calculate volumes of cylinders and cones in a practical context Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **$3\sqrt{3}$ or 5.2** cm and Shows work such as: $h^2 = 6^2 - 3^2$	1	1
2. a. Gives correct answer: **37.5π** or **117.8** cm^3 and Shows work such as: $\pi \times 2.5^2 \times 6$	1	
b. Gives correct answer: **45π** or **141.4** cm^3 and Shows work such as: $\pi \times 3^2 \times 3 + 2\pi \times 3$	2	
c. Gives correct answer: **15.6π** or **49** cm^3 and Shows work such as: $\pi \times 3 \times 5.2$	1	4
3. Gives correct answer: **3.5** cm	1	
Shows work such as: $141.4 \div 2 = 70.7$ volume of hemisphere = 56.5 volume of cylinder = 70.7 – 56.5 = 14.2 $14.2 = \pi \times 3^2 \times h$ $h = 0.5$ 3 + 0.5 =	2	3
Total Points		**8**

Geometry – 2008

Geometry Task 3 Pentagon

Student Task	Demonstrate geometrical knowledge and reasoning in a pentagon figure.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two- and three-dimensional shapes; develop mathematical arguments about geometric relationships; apply appropriate techniques, tools, and formulas to determine measurements.**
Core Idea 2 Mathematical Reasoning and Proofs	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples and indirect proof.** • Show mathematical reasoning in solutions in a variety of ways including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models. • Explain the logic inherent in a solution process. • Identify, formulate and confirm conjectures. • Establish the validity of geometric conjectures using deduction; prove theorems, and critique arguments made by others.

The mathematics of this task:

- Composing and decomposing shapes
- Knowing the degrees in a triangle, complementary and supplementary angles
- Knowing geometric properties like corresponding angles and sides
- Knowing properties of transversals for parallel lines
- Being able to make a complete proof for why something is true
- Not taking diagrams at face value, but using them with known facts to make and test conjectures

Based on teacher observations, this is what geometry students know and are able to do:

- Recognize similar figures
- Properties of isosceles triangles
- Degrees in a triangle

Areas of difficulty for geometry students:

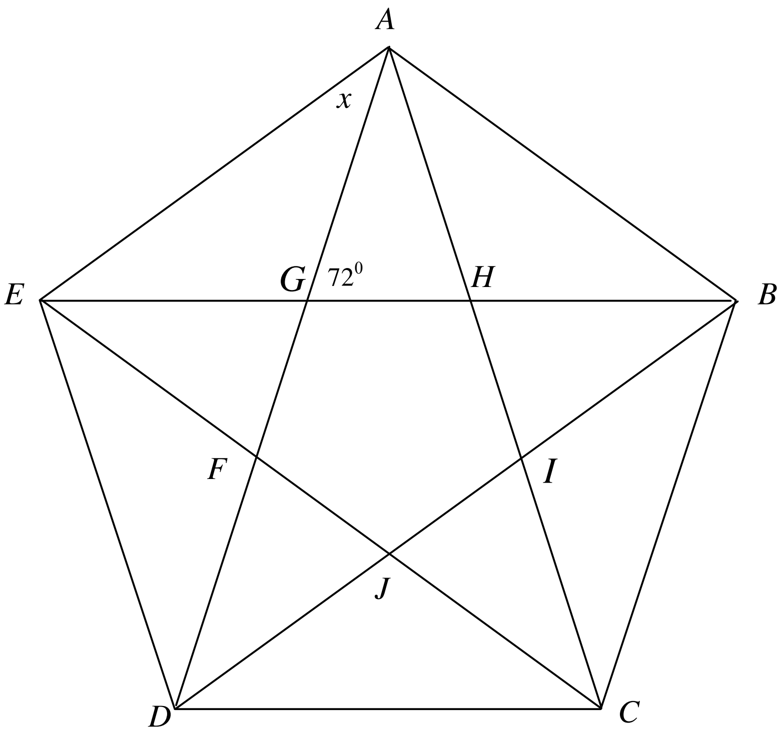
- How to use geometric proof to show why two triangles are similar
- How to complete a geometric argument, how much information is necessary
- How to make a justification not based on how something "looks"

Geometry – 2008

Pentagon

This problem gives you the chance to:
- demonstrate geometrical knowledge and reasoning in a pentagon figure

The diagram shows a regular pentagon ABCDE with all its diagonals shown.



1. What is a regular pentagon?

____

____

____

2. Find the measure of angle EAB?

____

____

3. Explain why the angle labeled x is 36^0.

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4. Using the letters in the diagram on the right, name the following shapes:

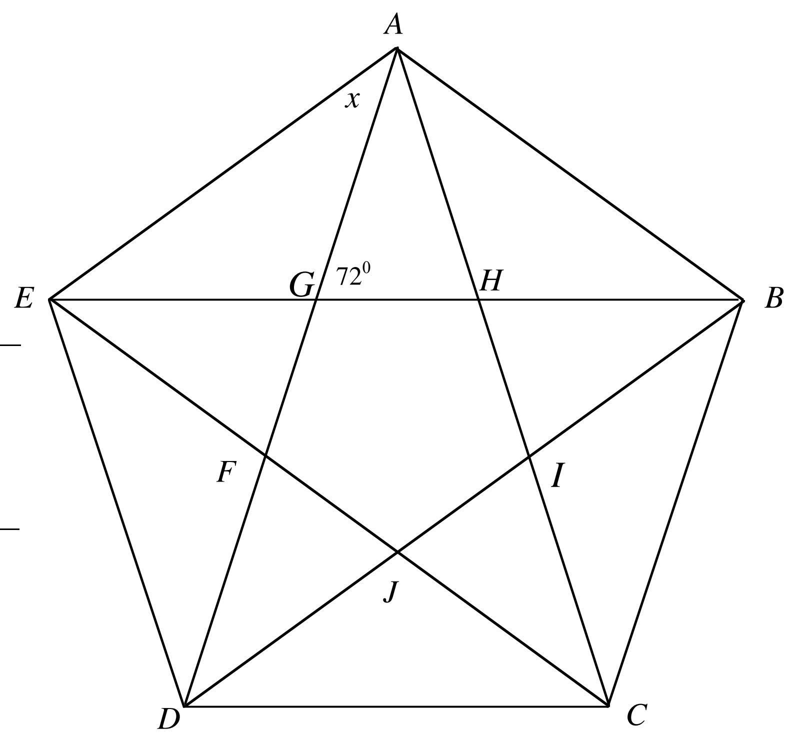
   a. a trapezoid

   ____

   b. an obtuse angled isosceles triangle

.

   ____



Geometry – 2008

5. a. Using the letters in the diagram on the right, name two triangles that are exactly the same shape but are different in size.

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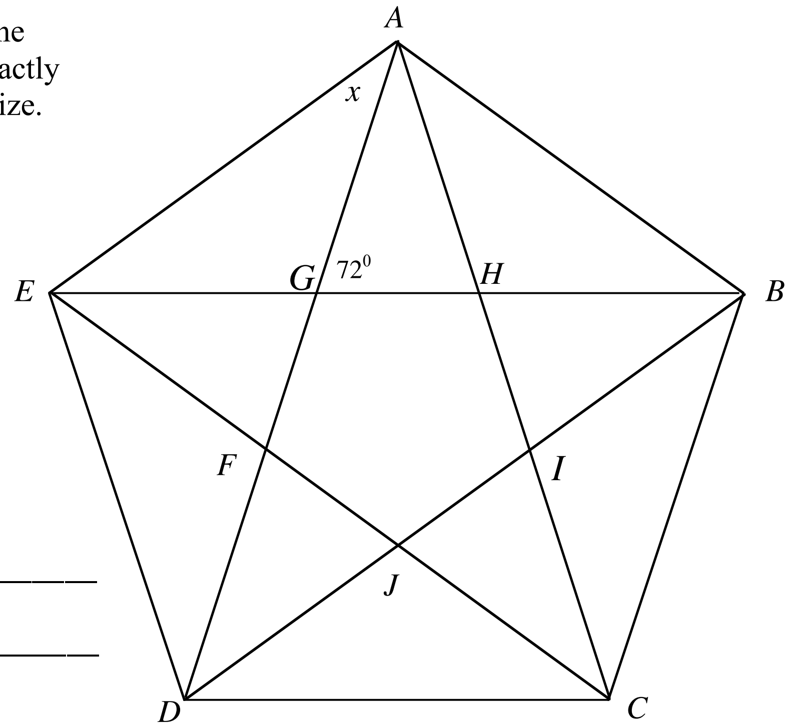

b. Explain how you know the triangles are similar.

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Geometry – 2008

Pentagon		Rubric
The core elements of performance required by this task are: • demonstrate geometrical knowledge and reasoning in a pentagon figure Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. A polygon with 5 equal sides and 5 equal angles	1	1
2. Gives correct answer: **108°**	1	1
3. Gives a correct explanation such as: Triangle DEA is isosceles (EA = ED) angle x = (180 – 108)°/2 = 36°	1	1
4.a. Names a trapezoid such as: EBCD b. Names an obtuse angled isosceles triangle such as: AEB	1 1	2
5.a Names two similar but not congruent triangles such as: AGH and ADC b. Gives a correct explanation such as: angle AGH =angle ADC because GH is parallel to DC angle ACD =angle AHG because GH is parallel to DC angle A is common to both triangles	1 2	3
Total Points		**8**

Geometry – 2008

Geometry Task 4 Triangles

Student Task	Work with areas of triangles within a triangle.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two- and three-dimensional shapes; develop mathematical arguments about geometric relationships; apply appropriate techniques, tools, and formulas to determine measurements.** • Understand and use formulas for the area, surface area, and volume of geometric figures.
Core Idea 2 Mathematical Reasoning and Proofs	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples and indirect proof.** • Show mathematical reasoning in solutions in a variety of ways including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models. • Explain the logic inherent in a solution process. • Identify, formulate and confirm conjectures. • Establish the validity of geometric conjectures using deduction; prove theorems, and critique arguments made by others.

The mathematics of the task:
- Understand the parts of a triangle by identifying base and height
- Understand the area formula and use it to present a reasoned argument about the relative size of nested triangles
- Use logical reasoning to make a complete justification
- Matching corresponding features to continue an argument about area
- Using a diagram to aid in problem solving, making necessary marks to help make sense of the situation
- Add, subtract and multiply fractions

Based on teacher observations, this is what geometry students knew and were able to do:
- Calculating with fractions

Areas of difficulty for geometry students:
- Identifying the correct height
- Applying the area formula as part of an argument
- Understanding that similarity does not apply if one side stays the same
- Distinguishing between restating the given information or proof statement and providing supporting evidence
- Making a complete justification

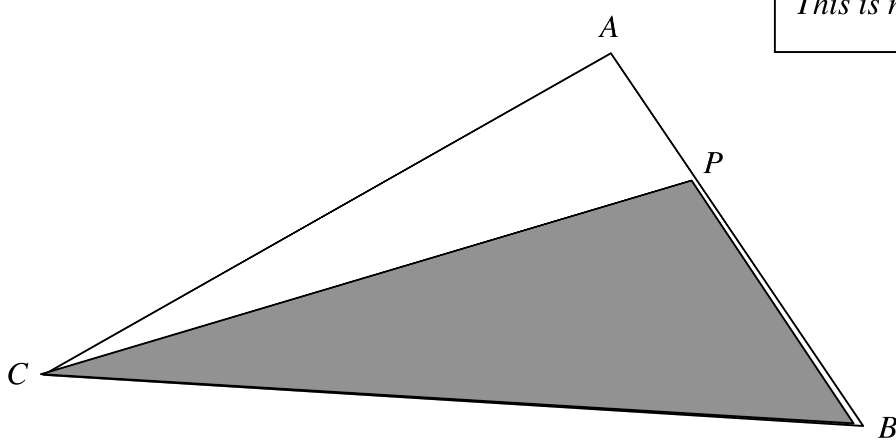
Geometry 2008

Triangles

This problem gives you the chance to:
• work with areas of triangles within a triangle

In triangle *ABC*, the point *P* is one third of the way from *A* to *B*.

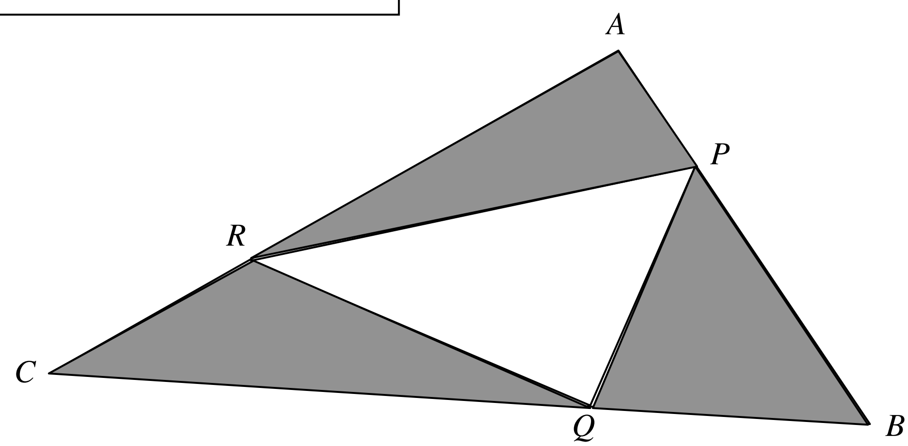
This is not an accurate diagram



1. Explain why the area of the shaded triangle is $\frac{2}{3}$ of the area of triangle *ABC*.

Geometry 2008

This is not an accurate diagram



Two new points, Q and R, have been added to triangle ABC.

Q is one third of the way from B to C and R is one third of the way from C to A.

2. Explain why the area of shaded triangle PBQ is $\frac{1}{3}$ of $\frac{2}{3}$ of the area of triangle ABC.
(Use your answer to Question 1 if it helps.)

3. What fraction of the area of triangle ABC is the area of the unshaded triangle PQR?
(Use your answer to Question 2a if it helps.)

Explain your reasoning.

8

Geometry 2008

Triangles		Rubric
The core elements of performance required by this task are: • work with areas of triangles within a triangle Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Provides a convincing explanation such as: The area of shaded triangle is 2/3 AB x h = 2/3 area triangle ABC	2	2
2. Provides a convincing explanation such as: Area CPB = 1/2 CB x h Area QPB = 1/2 QB x h but QB = 1/3 CB = 1/2 x 1/3 CB x h = 1/3 area CPB but area CPB = 2/3 area ACB so area QPB = 1/3 x 2/3 area of ACB	3	3
3. Gives correct answer: **1/3**	1	
Gives correct explanation such as: Using work similar to question #2, the area of each shaded triangle is 1/3 x 2/3 x area ACB ∴ the white area = 1 – 3 x 1/3 x 2/3 = 1/3 area ACB	2	
Partial credit Some reasonable explanation.	(1)	3
Total Points		**8**

Geometry 2008

Geometry Task 5 Circle and Squares

Student Task	Calculate ratios of nested circles and squares.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two- and three-dimensional shapes; develop mathematical arguments about geometric relationships; apply appropriate techniques, tools, and formulas to determine measurements.**
Core Idea 2 Mathematical Reasoning and Proofs	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples and indirect proof.** • Show mathematical reasoning in solutions in a variety of ways including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models. • Identify, formulate and confirm conjectures. • Establish the validity of geometric conjectures using deduction; prove theorems, and critique arguments made by others.

The mathematics of this task:
- Reading and interpreting a diagram
- Using Pythagorean theorem to find the lengths of the legs given a hypotenuse
- Working with squares and square roots
- Finding area of squares and circles
- Making ratios

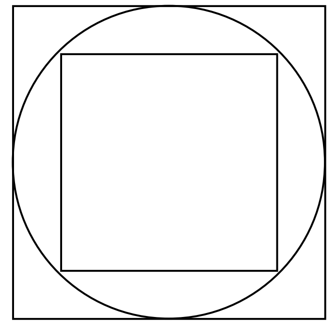
Based on teacher observations, this is what geometry students knew and were able to do:
- Find the side length of the large square
- Use Pythagorean theorem to find the side length of the small square

Areas of difficulty for geometry students:
- Using area to form ratios rather than side lengths
- Finding the size of the smaller radius

Geometry – 2008

Circle and Squares

This problem gives you the chance to:
• calculate ratios of nested circles and squares



This diagram shows a circle with one square inside and one square outside.

The circle has radius *r* inches.

1. Write down the side of the large square in terms of *r*. _________ inches

2. Find the side of the small square in terms of *r*. _________ inches
   Show your work.

3. What is the ratio of the areas of the two squares? ____________________
   Show your work.

4. Draw a second circle inscribed inside the small square.
   Find the ratio of the areas of the two circles. ____________________
   Show your work.

8

Geometry – 2008

Circles and Squares		Rubric
The core elements of performance required by this task are: • calculate ratios of nested circles and squares Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **2r**	1	1
2. Gives correct answer: $\sqrt{2}$**r** or 1.4r Uses the Pythagorean rule or trig ratios	1 1	2
3. Gives correct answer: **2** or 1/2 Shows work such as: $(2r)^2 \div (\sqrt{2}r)^2$	1 1	2
4. Gives correct answer: **2** or 1/2 Shows some correct work such as: the radius of the small circle is $r/\sqrt{2}$ the areas of the large and small circles are πr^2 and $\pi r^2/2$	1 2	3
Total Points		**8**

Geometry – 2008

Geometry Task 1 Triangles

Student Task	Show understanding of geometry and explain reasoning in a problem situation.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two-dimensional geometric shapes; develop mathematical arguments about geometric relationships; and apply appropriate techniques, tools, and formulas to determine measurements.**

Mathematics of the task:

- Reasoning about interior and exterior angles
- Understanding that the sum of the exterior angles for a polygon is always 360°
- Reasoning that the interior angles of a polygon are always a multiple 180°
- Developing a convincing argument or justification

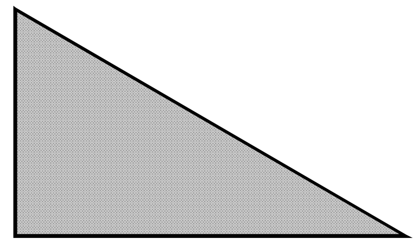
Strategies used by successful students:

- 50% used exterior angles 360/30
- 34% used the formula- 180 (n-2)/n
- 6% used the interior angles 1800/12 = 150

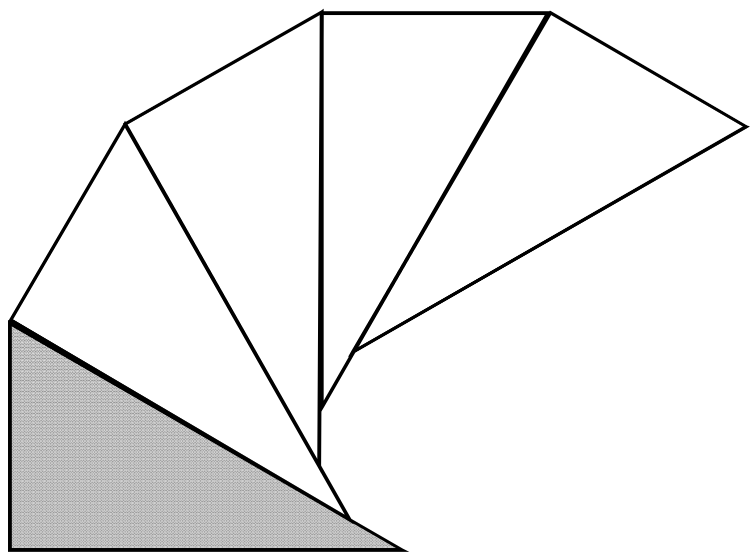
Triangles

This problem gives you the chance to:
• show your understanding of geometry
• explain your reasoning

This diagram shows a right triangle with angles of 60° and 30°.



The second diagram shows five copies of the same triangle fitted together.
If this continues, a regular polygon with a hole in the middle will be formed.



Use the angles of the triangle to calculate the number of sides the regular polygon will have.
Explain all your reasoning carefully.

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5

2009 Rubrics Grade 10

Triangles		Rubric
The core elements of performance required by this task are: • show your understanding of geometry • explain your reasoning Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: **12 sides**	2	
Gives correct explanation which may involve the external angles of the polygon.	3	
Partial credit Incomplete explanation	(1)	5
Total Points		**5**

Geometry

Student Task	Work with the volumes of pyramids and hemispheres.
Core Idea 4 Geometry and Measurement	**Apply appropriate techniques, tools, and formulas to determine measurements.** • Understand and use formulas for the area, surface area, and volume of geometric figures, including spheres and cylinders. • Visualize three-dimensional objects from different perspectives and analyze their cross sections.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Solve equations involving radicals and exponents in contextualized problems such as use of Pythagorean Theorem.

Mathematics of this task:

- Recognizing the base in a pyramid
- Being able to decompose a formula into smaller parts to derive the needed numbers such as area of the base
- Understanding that the height of a triangle is not always equal to the side length
- Being able to use Pythagorean theorem in a practical application
- Understanding equality to set up an equation to make the volume of one figure equal to the volume of another
- Understanding circles and spheres, recognizing the difference between radius and diameter, a half sphere and a whole sphere

Based on teacher observation, this is what geometry students know and are able to do:

- Use the formula the find the volume of a sphere
- Find the volume of a pyramid with a square base

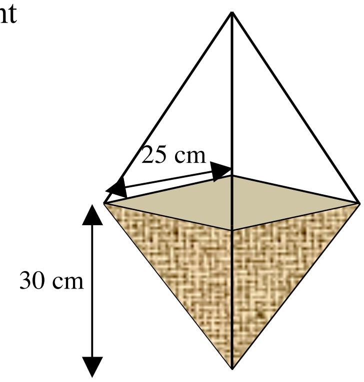
Areas of difficulty for geometry students:

- Confusing the base of the square for the area of the base in the pyramid
- Understanding that the side length is not the height of an equilateral triangle
- Understanding that the area of a 3-dimensional figure requires an area of the base times the height
- Using the height from the base triangle as the height for the pyramid
- Using the area from part 1 instead of the volume from part 1 to solve in part 2
- Forgetting that the formula was for the area of a sphere and that the figure was a hemisphere
- Squaring the radius when finding the volume of a sphere instead of cubing the radius

Hanging Baskets

This problem gives you the chance to:
- work with the volumes of pyramids and hemispheres

Hugo sells hanging baskets. They have different shapes and sizes.
Hugo needs to know their volume so that he can sell the correct amount of potting compost to fill them.

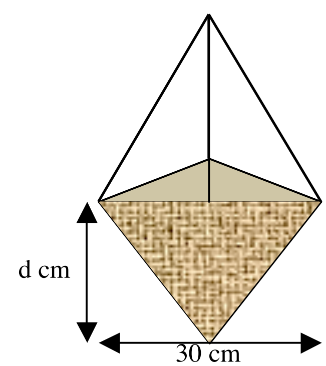


1. There are pyramid shaped hanging baskets with a square opening at the top.
   The square has sides of 25 cm and the basket is 30 cm deep.

Volume of a pyramid = $^1/_3$ *area of base* x *height*

   Calculate the volume of this basket.
   Show your work.

   ___________________cm^3



2. Hugo makes a tetrahedron shaped basket with the **same volume** as the square based pyramid.
   He decides to make it with an opening that is an equilateral triangle with sides that measure 30 cm.

   Find the area of the equilateral triangle. _____________ cm^2

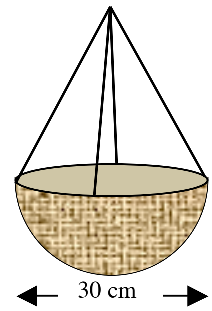
   How deep will this basket have to be? ___________________cm
   Show how you figured it out.

Geometry

Hanging Baskets

3. Hugo also sells hemispherical baskets with a diameter of 30 centimeters.

Volume of a sphere = $^4/_3\pi r^3$



Calculate the volume of this basket. __________________ cm^3
Show your work.

9

Geometry

Hanging Baskets

Hanging Baskets		Rubric
• The core elements of performance required by this task are: • work with volumes of pyramids and hemispheres Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **6250** Shows correct work such as: 25^2 x 30 / 3	1 1	2
2. Gives correct answer: 390 ± 1 Gives correct answer: approx 48 Shows correct work such as: 390/3 = 130 6250 / 130	2 1 ft 1 ft	4
3. Gives correct answer: **7069 or 2250π** *Partial credit:* 14137 or 4500π, 56549 or 18000π Shows work such as: 2/3 π 15^3 or 4/3 π 15^3 or 2/3 π 30^3	2 (1) 1	3
Total Points		**9**

Geometry

Geometry Task 3 Pentagon

Student Task	Show understanding of geometry and write mathematical proofs.
Core Idea 2 Mathematical Reasoning and Proof	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples and indirect proof.** • Show mathematical reasoning in solutions in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models. • Explain the logic inherent in a solution process. • Identify, formulate and confirm conjectures. • Establish the validity of geometric conjectures using deduction; prove theorems, and critique arguments made by others.

Mathematics of this task:

- Show why the interior angle of a regular pentagon is 108°
- Decompose a complex figure
- Make a convincing justification about similarity and congruency using given information
- Understand the difference between what "looks to be true" and what needs to be proven

Based on teacher observation, this is what geometry students know and are able to do:

- Find the interior angle of a regular figure
- Know the theorems for similarity and congruency
- Identify corresponding parts in similar and congruent figures
- Use properties of supplementary angles
- Use reflexive property

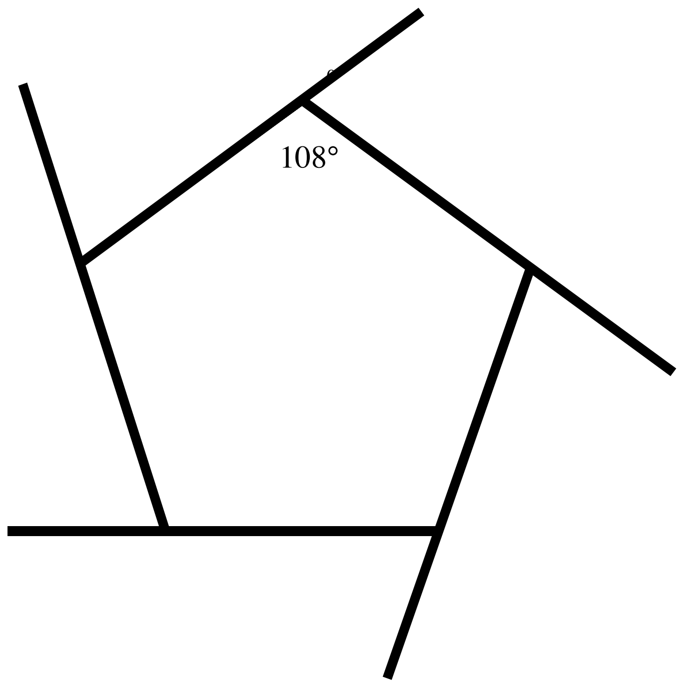
Areas of difficulty for geometry students:

- Assuming sides of a triangle are isosceles
- Assuming parallelism
- Assuming angles within a figure are divided equally
- Assuming properties of diagonals with making a justification
- Using properties of similarity or congruency to make the proof of similarity or congruency, e.g. if the triangles are congruent the sides are equal, the sides are equal because its congruent, therefore its congruent

Pentagon

This problem gives you the chance to:
• show your understanding of geometry
• write mathematical proofs

The diagram shows a regular pentagon.



1. Explain why each inside angle of the regular pentagon is 108°.

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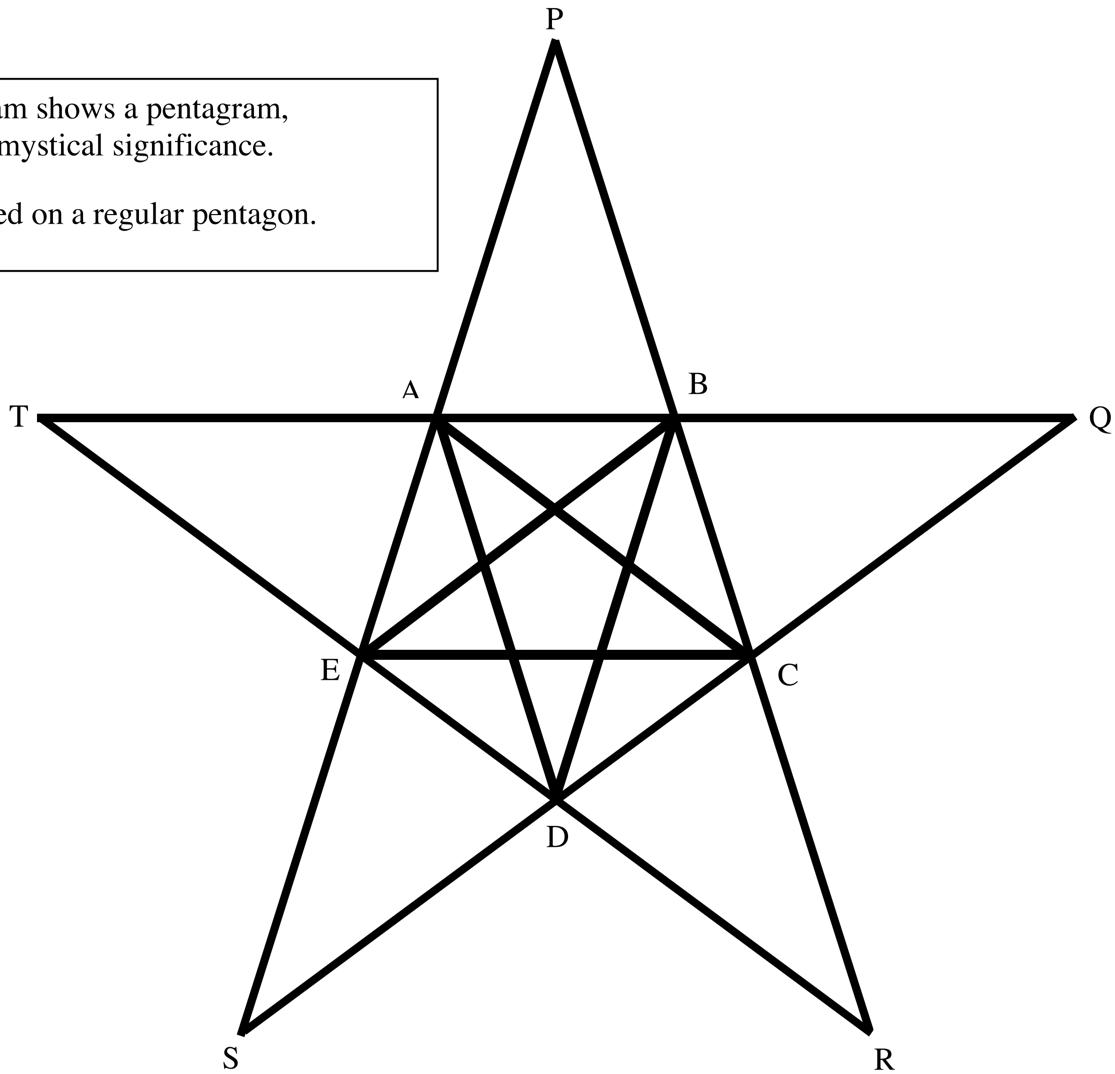
Geometry

Pentagon

This diagram shows a pentagram, a shape of mystical significance.

It is centered on a regular pentagon.



2. Show that triangle PCE is similar to triangle ACE.

3. Prove that triangle PAB is congruent to triangle DAB.

8

Pentagon		Rubric
The core elements of performance required by this task are: • understanding of geometrical situations • construction of mathematical proofs and explanations Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct explanation such as: Recognition that sum of exterior angles is 360^0 Calculation of each exterior angle (72^0) Calculation of each interior angle ($180^0 - 72^0$) Accept alternative methods such as: $\frac{(5-2)180}{5} = 108$ *Partial credit* Incomplete explanation.	3 (2)	3
2. Determines the measures of the angles in each triangle. (72^0, 72^0, 36^0) Recognition that triangles are therefore similar. *Partial credit* Incomplete explanation	3 (2)	3
3. Recognition and explanation of similarity of triangles Explanation that as triangles have a corresponding common side, they must be congruent	2	2
Total Points		**8**

Geometry Task 4 Circle Pattern

Student Task	Use algebra patterns to explore a geometric situation. Explore fractions in context
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Solve equations involving radicals and exponents in contextualized problems.
Core Idea 2 Mathematical Reasoning	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples and indirect proof.**
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two-dimensional geometric shapes, develop mathematical arguments about geometric relationships; and apply appropriate techniques, tools, and formulas to determine measurements.**

Mathematics of the task:

- Use area formula to make a generalization for any size circle
- Notice a pattern about area using fractional parts
- Be able to look at features of a pattern to make a generalization about how it grows

Based on teacher observations, this is what geometry students know and are able to do:

- Find the black fractional area for a circle with 4 small black circles
- Fill in some of the numbers in the table
- Find some key attributes of the pattern

Areas of difficulty for geometry students:

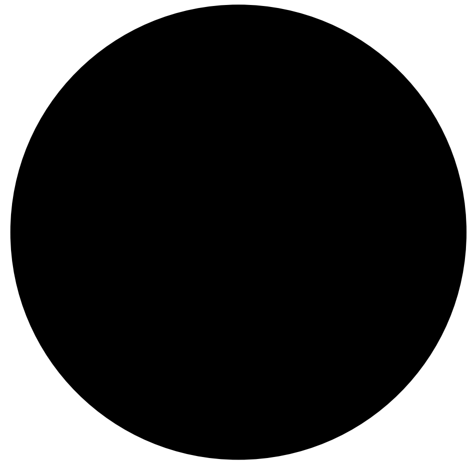
- Using the area formula to prove that in the first case, the white circles are half the area of the black circle
- Completing all the lines of the pattern, often because of generalizing the pattern too quickly
- Generalizing the growth pattern for the areas of the circles

Circle Pattern

This problem gives you the chance to:
• explore fractions in context

Here is a developing circle pattern.

Here is one black circle.



Two white circles of half the radius have been added to the diagram.

1. Show that the fraction of the diagram that is now black
    is one half.

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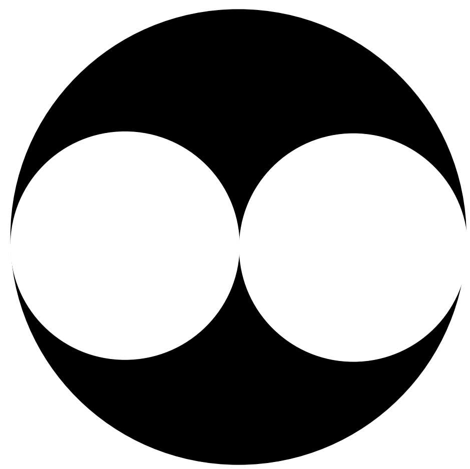

Four black circles have now been added.

2. What fraction of the diagram is now black?

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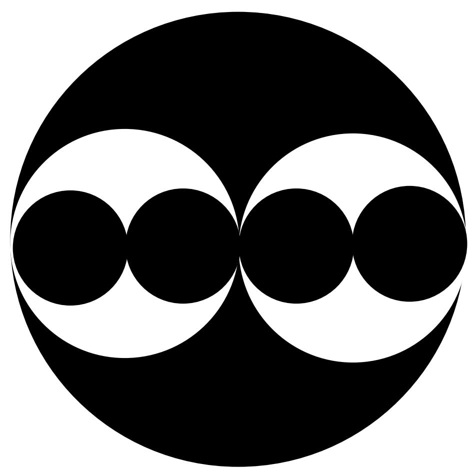
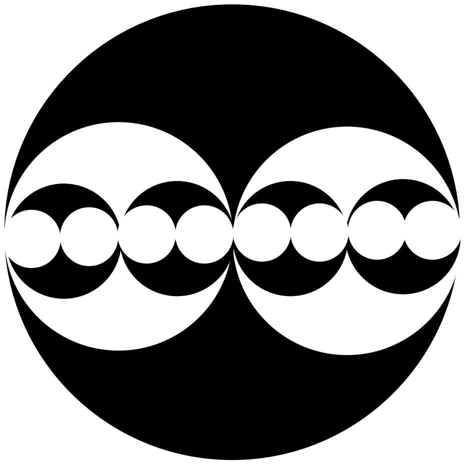
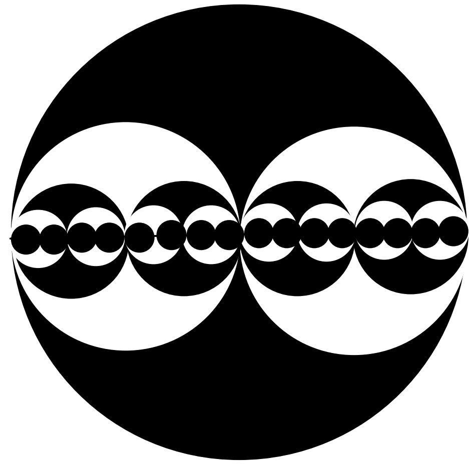


Geometry

Circle Pattern

3. Fill in the table to show what happens as the pattern continues.





Pattern	Black fraction	White fraction
One black circle	1	0
Two white circles	$\frac{1}{2}$	$\frac{1}{2}$
Four black circles		
Eight white circles		
Sixteen black circles		

4. Write a description of what is happening to the black and white fractions as the pattern continues.

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9

Geometry Circle Pattern

Circle Pattern		Rubric
• The core elements of performance required by this task are: • explore fractions in context explore f Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct explanation such as: Let radius white circle be r, then area = πr^2 Radius black circle is 2r, then area = 4 πr^2 Area of two white circles is 2 πr^2	2	2
2. Gives correct answer: **3/4**	2	2
3. Gives correct answers: **3/4, 1/4, 5/8, 3/8, 11/16, 5/16** *Partial credit* 4 or 3 correct two points 2 correct one point	3 (2) (1)	3
4. Gives correct explanation such as: Each time a half of the previous fraction is added or subtracted from the black fraction. (The limit of the black fraction is 2/3.) *Partial credit* For a partially correct explanation that either addresses change by half or the oscillating adding or subtracting.	2 (1)	2
Total Points		**9**

Geometry

Geometry Task 5 Floor Pattern

Student Task	Show understanding of geometry and explain geometrical reasoning.
Core Idea 4 Geometry and Measurement	**Analyze characteristics and properties of two-dimensional geometric shapes, develop mathematical arguments about geometric relationships; and apply appropriate techniques, tools, and formulas to determine measurements.**
Core Idea 2 Mathematical Reasoning	**Employ forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures and using counter examples and indirect proof.** • Show mathematical reasoning in solutions in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models. • Identify, formulate and confirm conjectures. • Use synthetic, coordinate, and/or transformational geometry in direct or indirect proof of geometric relationships. • Establish the validity of geometric conjectures using deduction; prove theorems, and critique argument made by others.

Mathematics of the task:
- Use geometric properties of circles, squares, and kites to prove the angles of a quadrilateral
- Understand theorems needed to prove congruent triangles
- Understand theorems needed to prove a quadrilateral is a parallelogram
- Develop a justified chain of reasoning to support that a shape is a parallelogram
- Compose and decompose a complex figure

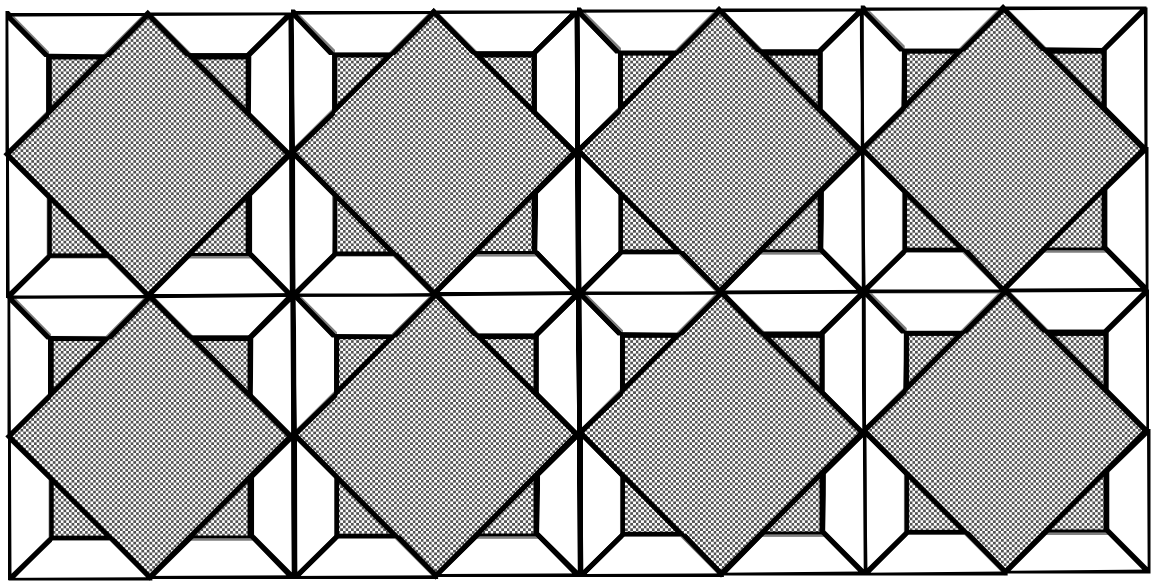
Based on teacher observation, this is what geometry students know and are able to do:
- Find 4 angles of a quadrilateral
- Explain how the angle measure was derived
- Use congruence to show that AB is equal to DC
- Understand that opposites of a quadrilateral need to be equal in order for the shape to be a parallelogram

Areas of difficulty for geometry students:
- Not making assumptions about congruency of triangles without giving the angle measure
- Taking AD = BC as a given instead of something that needs to be proved
- Assuming that because AB =DC, that the sides must be parallel
- Not understanding the steps needed after AB=DC, stopping at that point

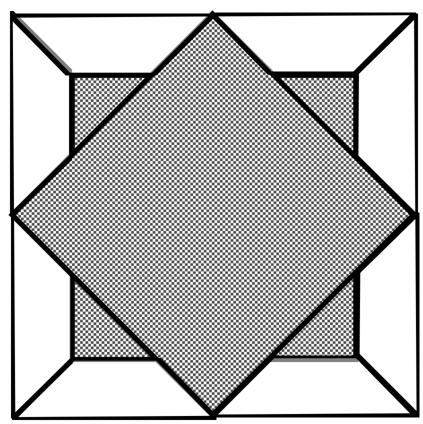
Floor Pattern

This problem gives you the chance to:
- show your understanding of geometry
- explain geometrical reasoning

The diagram shows a floor pattern.



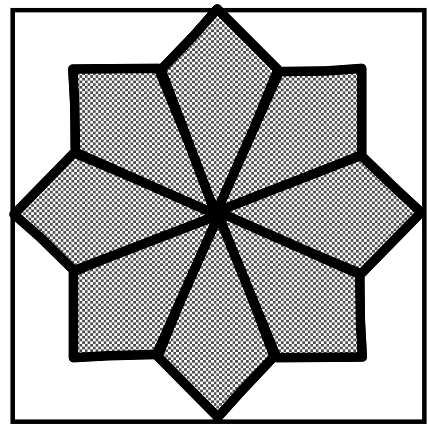
In the floor pattern, the shaded part is made by overlapping two equal squares.



The shaded shape can also be seen as a set of eight equal kites.

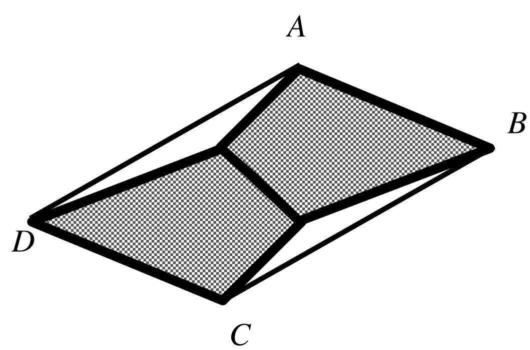
1. Find the measures of all four angles of the kites.

Explain how you obtained your answers.



2. Two of the kites can fit together to make a hexagon.

Prove that the quadrilateral ABCD is a parallelogram.



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Geometry

Floor Pattern

Floor Pattern		Rubric
• • The core elements of performance required by this task are: • show your understanding of geometry • explain geometrical reasoning • Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: **90°, 45°, 112.5°, 112.5°**	3 x 1	
Gives correct explanations such as: The 90° angle is the corner of a square. The 45° angle is 360÷8. The other two angles are equal and the angle sum is 360°.	2 x 1	5
2. AB = DC	1	
Gives correct explanation showing that ABCD is a parallelogram..	3	
Partial credit Incomplete explanation.	(1)	4
Total Points		**9**

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