Aiming High
A Countywide Commitment To Close The Achievement Gap For English Learners
Aspirando a lo Mejor
RESOURCE
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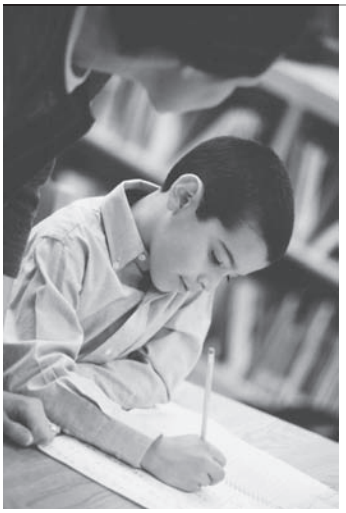


Making mathematics accessible to English Learners

Implementing research-based mathematical literacy practices

The study of mathematics can be difficult for students, but it is especially challenging for students who are learning English. Language barriers and misunderstandings can cause English Learners to struggle with even the most basic concepts in this content area. But when teachers understand the language and literacy demands specific to mathematics—and are able to apply a range of instructional strategies—they can help English Learners succeed in this important subject.

> With language minority students, teachers must attend not only to their cognitive development but also to the linguistic demands of mathematical language. The importance of language in mathematics instruction is often overlooked in the mistaken belief that mathematics is somehow independent of language proficiency. However, particularly with the increased emphasis placed on problem solving, command of mathematical language plays an important role in the development of mathematical ability (Anstrom, 1997).

As a gatekeeper for entrance into higher education, **mathematics has a greater impact on the advancement of English Learners than all other content areas except English itself**. Algebra filters out English Learners faster than any other mathematics course (Lager, 2003) and can act as a barrier to post-secondary education. Helping English Learners succeed in algebra is critical for schools working to close the achievement gap. Teachers must make mathematics more accessible to English Learners throughout *all* the grade levels in order to support this goal.

This publication is based on a presentation made at the 2005 ELL Administrator Conference hosted by the Sonoma County Office of Education (SCOE). Joan Easterday, Jane Escobedo, and Doreen Heath Lance were the presenters. As part of the Aiming High initiative, SCOE is providing this brief to teachers throughout Sonoma County.

Today, mathematics classrooms are language-centered learning environments that require students to process information, perform calculations, and explain how to solve problems. Mathematics instruction for English Learners should not focus solely on numbers; these students must also be supported in building linguistic flexibility and explicitly learning the vocabulary unique to this content area. The linguistic aspect of mathematics is crucial!

Even relatively simple mathematical problems can carry unexpected language demands.

There are a variety of instructional strategies that teachers can use to ensure greater understanding of mathematical skills and concepts among language minority students. This publication highlights some discipline-specific practices that are centered on three key components of mathematical learning:

Unpacking the problem,
Processing information in the problem, and
Summarizing the solution with evidence.

Unpacking the problem

In this phase of mathematics instruction, the teacher's role is to help students understand the problem and what's being asked. This process is sometimes known as "unpacking the problem." During the unpacking process, teachers guide students by:

- Making connections between previous learning and the unique features of the new problem,
- Calling out and defining the language terms and structures needed to communicate mathematical understanding of the problem, and
- Connecting students to what's being asked.

Even relatively simple mathematical problems can carry unexpected language demands. In order to better understand the complexity of mathematical problems, researchers at the University of Wisconsin-Madison and UCLA studied word problems that require addition, subtraction, multiplication, and division. They grouped addition and subtraction problems into 11 problem types and studied the strategies that students use to solve them.

One local example of how this research is helping teachers develop mathematical understanding and linguistic ability can be seen at **Kawana School** in the Bellevue Union School District. First-grade teachers in a lesson study group at Kawana realized that comparison word problems were proving mathematically difficult for their students because of the language involved. For example, problems that asked, "How many more than…?" led to misunderstandings because the word *more* triggered the suggestion of addition to students.

It's important for students to grasp the concept of comparison word problems in the early grades because this type of problem is prevalent across the grade levels, from kindergarten math through high school algebra. The following examples are typical of the comparison problems students are asked to solve in different grades:

- Sid takes 3 pictures on Monday and 9 pictures on Thursday. How many more pictures did he take on Thursday than on Monday? (First Grade)
- A small and large eagle poster cost $12 altogether. A large poster costs $4 more than a small poster. How much does each poster cost? (Third Grade)
- Tana's age is 6 years more than twice Mark's age. In two years, Tana will be three times as old as Mark. How old is Tana? (Sixth Grade)

To ensure student understanding of problems like these, teachers should analyze mathematical problems for vocabulary and language patterns. Kawana's first-grade teachers studied the language demands, then created the following response frames to highlight the mathematical language required for their comparison word problems.

There are ____ more.
____ has more than ____, but less than ____.
____ is the least and ____ is the most.
I think ____ is more/less than ____ because ____.

These frames were developed to help all students, but they are especially important tools for helping English Learners construct oral and written responses to the questions posed. To provide even more support, the first-grade teachers developed an activity that allowed them to scaffold instruction for English Learners and other students who were struggling with mathematics

concepts. This lesson was centered on the following comparison word problem:

> *Bill brought goldfish crackers to share with his class. Lupita took 5, Carlos took 2, and Juan took 3. How many more goldfish does Lupita have than Juan?*

The teachers began the lesson by asking students what the word *compare* means and what they could compare in their lives. Students talked about comparing books, measuring the length and weight. Teachers wrote "comparing words" on the board and students discussed the meaning of the words.

Next, the teachers read the word problem to the class and distributed goldfish crackers. They had the students recreate the problem in person—students role-played Lupita, Carlos, Juan, and Bill—and use a graph to illustrate the problem. The graph organized the quantities in a grid so students could visually compare the amounts. Teachers used the graphic representation to reinforce important information and provide an alternate representation for students.

The students discussed what they noticed about the graph with a partner, then shared their observations with the whole class. Teachers recorded their statements in large speech bubbles and highlighted the comparing words students had identified.

Hildardo: I see 6 goldfish.

Michael: Lupita has 6 goldfish.

Martha: Lupita has **the most**.

Christian: Carlos has **the least**.

Elias: Lupita has **the most** of everybody.

Andrew: Juan has **more than** Carlos and **less than** Lupita.

Using the response frames and speech bubbles, the teachers connected students' spoken words to written text, linking informal speech patterns to the more formal, academic patterns typical in math textbooks.

High school students also need help connecting to the language of mathematics. An Algebra I class at **Roseland University Prep** was studying rational expressions. In order to build background, motivation, and context for the many English Learners in this class, the teacher chose to anchor this complex mathematical concept in something that every student could relate to—homecoming week. The students were given the task of planning how much to charge for tickets to the upcoming homecoming week activities. First, though, the matter of mathematical language had to be addressed.

The teacher pointed out that, in using rational expressions, students would be converting words and statements into mathematical equations that could be solved. To emphasize this idea, the teacher suggested that the class create a compare/contrast chart. Students worked together to complete this chart, identifying the characteristics of and defining two important target vocabulary words, *expression* and *equation*.

The teacher then explained that a mathematical expression is similar to a verbal expression—it simply makes a statement—while an equation is a number sentence with two equal parts. The class discussed the mathematical expression $(2x + 3) \div 5$ and compared it

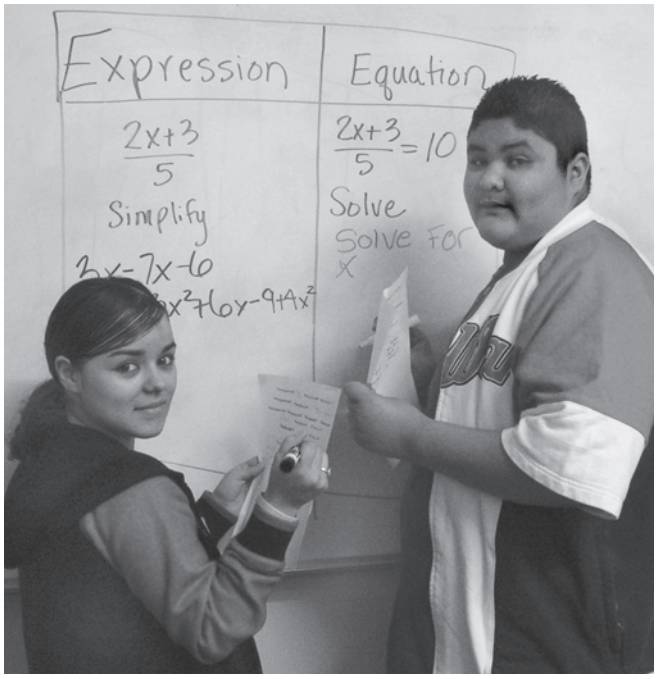
Words that compare

- Most
- More than
- Words with –er
- Least
- Less than
- Words with –est

Idea: Make a mathematical word bank on the wall or have students record the words in their notebooks. Students can refer to this vocabulary when they need to express their understanding during assessments and reporting exercises.

to the equation (2x + 3) ÷ 5 = 10. Students determined that the x in the expression could be any number, while the x in the equation must be 23.5.

As an extension activity, the students wrote definitions of expression, equation, and other target words in their notebooks. They wrote these definitions in their own words, illustrated their understanding of the word, provided an example, and wrote a sentence using the word. This activity served two purposes. It strengthened student understanding of the concepts represented by the target vocabulary and it resulted in a mathematical dictionary students could refer to during future work sessions.



Processing information in the problem

Once students understand what's being asked in a problem, they can begin to solve it. In the solving stage, teachers should use a variety of instructional strategies to guide students in processing information related to a mathematical problem, developing understanding of the information, and communicating their understanding. Visual representations, pair and small group discussions, teacher prompts, whole-group sharing, guided reporting, and multiple representations are among the strategies that have proven effective in this phase of instruction.

In the first-grade classrooms at Kawana School, students worked in pairs to solve the goldfish problem. They recorded their solutions on the board and the class discussed the different ways students figured out the answer. The teachers employed strategies to help students visualize the solution, such as drawing pictures, matching crackers and counting what was left, counting backwards to the matched amount, and subtraction algorithms.

When the class discussed the problem, here's how the teacher helped an EL student talk about what he learned.

Teacher: I see you wrote Lupita and Juan and you drew 6 fish in a circle. You crossed some out. What were you showing by crossing them out?

Christian: They both have the same amount. Lupita take away 3.

Teacher: So, how would I write that as an equation?

Christian: 7, no, 6 – 3 = 3

How to teach mathematical vocabulary

1. Identify key mathematical words (words that are essential to understanding)—*rate, slope, more, less, least, most.*
2. Identify key concept words that can be used across the curriculum—*compare, contrast, evaluate, difference, disparity.*
3. Pick 3-5 mathematical words and 2-3 concept words.
4. Introduce each word and chorally repeat.
5. Give each word a "student-friendly definition" in addition to the one found in the dictionary. Ask students to restate the definition in their own words.
6. Ask students to illustrate the concept the word represents.
7. Give students real-life examples of how the word is used by asking questions: Which is more, 25 jelly beans or 10 jelly beans? Have students write their own sentences using the word.

★ **Idea:** Have students develop a math dictionary with important words (see sample below).

Word	*Definition*	*Illustration*	*Example*

Teacher: You said they both have the same.

Christian: They both have the same fish.

Teacher: What do they have that is the same?

Christian: They both have 3 fish.

Teacher: They both have 1, 2, 3 and 1, 2, 3. That's the same about them. What's different?

Christian: Lupita has 6 more?

Teacher: 6 more? You said they each have 3 and Lupita has 6 more?

Christian: No, 3 more.

Teacher: Oh, she has 3 more. You told me 6 – 3 = 3 and you crossed out 3. What answer did you get?

Christian: 3 more.

The teacher moved Christian through the process of identifying what was the same and what was different, then counting the difference. His language changed from "same" to "same fish" to "they both have 3 fish" and, finally, to the answer, "3 more." By reprocessing information, the student was able to "show his thinking." The teacher guided him to clarify what was being asked and to extract the information that led to a solution.

When guiding students in discussions about the complexities of mathematical words, phrases, and symbols, teachers can help students clarify their thoughts and understandings. For students who are learning English and math simultaneously, it's especially important to give them reasoning and language tools—tools that allow them to explore mathematical ideas and the language that enables them to talk about those ideas. In addition, English Learners need multiple opportunities to work through mathematical problems and to "rehearse" using academic vocabulary.

Teachers should analyze mathematical problems for vocabulary and lanaguage patterns.

Back at Roseland University Prep, the Algebra I students were working to determine how much they needed to charge for homecoming activity tickets. The teacher explained that the cost of the hall rental was $150, plus a $15 cleaning fee, and that 10 people would attend on Monday and 15 on Tuesday. To ensure that students understood the problem, the teacher illustrated each element on the board (hall, cleaning fee, and number of people) as she talked through the problem. Then, she asked students to write an *equation* for the problem.

Later, students worked in pairs to develop progressively more complex mathematical *expressions*, replacing the $150 cost of the rental hall with the variable P. The teacher confirmed student understanding by asking them to repeat each expression and explain what the elements represented.

These examples highlight some key strategies for helping English Learners process and solve mathematical problems:

- **The teachers' demonstrations used both language and visuals.** This increases the likelihood that students will be successful learning mathematical concepts and procedures.
- **Students had multiple practice opportunities** to process important

Reprocessing strategies

Researchers suggest that teachers can stretch language skills and thinking about math among English Learners by encouraging them to talk about math concepts, problems, and solutions. Teachers can ask EL students to:

- Restate another student's statement
  Can you say that another way?
- Apply their reasoning to another student's solution
  Do you agree/disagree?
- Write or draw about their mathematical understandings
  Can you draw a chart to explain your answer?

Idea: Use math journals to help you see how students are using academic language and to determine what steps are needed to promote growth toward academic and linguistic proficiency.

ideas, vocabulary, and language structures in different ways.

- **Students were grouped in teams** to clarify, practice, and rehearse the language needed to report their findings to the class. Through teamwork, students also gain support for their learning from their peers.
- **Associations were made with other information** to solidify learning and move it to permanent memory.
- **Additional detail was added** once students demonstrated that they had grasped key concepts and become proficient using academic terms.

Summarizing a solution with evidence

In this phase of instruction, teachers guide students to explain how they found a solution. As they summarize their learning, students restate the procedural and declarative knowledge they've gained during a problem-solving lesson.

Teachers should encourage students to use pictures, manipulatives, and language to provide evidence of how and why they support a particular solution. When given the opportunity to present information in a variety of ways, students are more likely to understand and extend their use of academic language. This is especially important for English Learners. By carefully guiding EL students in the summarizing process, teachers can sharpen students' use of precise, academic mathematical language and move them away from language that is unclear, informal, or vague.

English Learners need multiple opportunities to work through mathematical problems and to "rehearse" using academic vocabulary.

In the summarizing phase of their lesson, the Roseland University Prep students were placed in small groups. Students worked together to write an expression to represent what it would cost to attend homecoming activities each night of the week. They also explained how they developed this mathematical expression and why it would be useful.

The student groups presented their solutions in a variety of ways. One group of limited English proficient students developed a PowerPoint presentation, using their math dictionaries to choose and clarify vocabulary. The presentation included bulleted points and graphics to explain the steps they took to find the answer. By using PowerPoint, these students had an easier time selecting, processing, and developing the language for their presentation and demonstrating their understanding of the concepts involved.

Another group of students used a transparency to share their mathematical expression with the class. Each member of the group explained one step in the development of their solution. Still another group of students presented in both English and Spanish, connecting their understanding of mathematical concepts and vocabulary in their first language to their second. There were also a few students who reported directly to the teacher in one-on-one discussions, lowering their anxiety for using new language in front of a large group.

Reporting by first-grade students is understandably more basic. At Kawana School, the first-graders reported their findings by counting with their fingers, drawing pictures, circling and labeling the fish that represented "more than," using a graph to demonstrate who had more, and writing the equation of $6 - 3 = 3$. Students with less fluency in English communicated with drawings and labels, while more fluent

Assessment options

- Emphasize **speaking** and **listening** for students at the beginning and early intermediate levels
- Emphasize **reading** and **writing** for students at the intermediate grades and above
- Use **multiple representations** such as drawing, manipulatives, short written answers, and partner activities
- Emphasize **thinking processes** so students describe why and how they did what they did through writing, speaking, and drawing

Teachers should use a variety of instructional strategies to guide students in processing information related to a mathematical problem.

students used sentences with target vocabulary and comparative sentence forms.

As seen in both the first-grade and high school classes, the differentiation of reporting allowed the entire class to work toward competence on a grade-level standard, while also providing support to students with less language proficiency. These lessons were successful because they included clear mathematical goals, explicit teaching of target vocabulary, structured interaction in small and large groups, and teacher-guided reporting. By using these techniques, teachers can be assured that *all students* have access to important mathematical concepts and the opportunity to expand academic language proficiency.

In order for students to be able to think, talk, and write like mathematicians, they must be apprenticed in the discourse of mathematics. This requires that teachers model, make strategies explicit, and provide the tools students need to become active constructors of knowledge. ■

Steps to solving and paraphrasing mathematical problems

★ **Paraphrasing** allows students to think and talk about their understanding of math problems. In this activity, students work in pairs to review a problem, talk about their observations, and collaborate to find a solution. This is a particularly effective strategy for English Learners because it gives them the opportunity to use academic vocabulary. Here's an example of how it works.

1. **Students** are grouped in pairs and given a problem: *On Sunday, Paul walked 3 times as far as he did on Saturday. On Saturday, he walked half as far as he did on Friday. If Paul walked 2.5 miles on Friday, how many miles did he walk altogether?*
2. **Teacher** teaches vocabulary words and phrases that are important to understanding: *altogether, times, half, as far as.*
3. **Students** take turns picking words and telling their partners what they mean: *"Altogether" means put everything together.*
4. **Teacher** reviews the question being asked: *How many miles did he walk altogether?*
5. **Teacher** presents any additional questions: *How far did he walk on Saturday? How far did he walk on Sunday?*
6. **Working in pairs**, students repeat the questions and paraphrase them in their own words: *It is asking how far he walked in all three days.*
7. **Teacher** reviews important information: *Sunday he walks 3 times Saturday; Saturday he walks half of Friday; Friday he walks 2.5 miles.*
8. **Students** tell their partners the important information: *He walked 2.5 miles on Friday, half as far on Saturday, and 3 times Saturday's miles on Sunday.*
9. **Student pairs** plan a strategy for solving the problem, then share their strategy with another pair: *We will cut pieces of paper to show the distances for each day, then put them in a line to see how far he walked altogether.*
10. **Students** work in pairs to implement their strategy, then write equations in words and numbers to show what they learned:

    Friday divided by two = Saturday (2.5 ÷ 2 = _____)

    Saturday times 3 = Sunday (___ x 3 = ___)

    Friday plus Saturday plus Sunday = the answer
11. **Student pairs** present their solution and tell why they solved it this way: *The solution is 7.5 miles because we used 2.5 for Friday, 1.25 for Saturday, and 3.75 for Sunday, then added them together.* ■

Teacher-guided reporting

When students are asked to report to the class about what they've done or learned, teachers can provide scaffolding by guiding this process. As teachers clarify, question, and provide models for the speaker, they work collaboratively with the student to build language and reporting skills. Here's how it works.

The teacher begins by presenting an open-ended prompt to the student. For example:

Tell us what you learned.
Tell us about what you did.
What did you find out?
What would you like to tell us about?
What did you find most interesting?

Teachers can slow down the overall pace of the dialogue, giving English Learners time to process what they want to say and how they are saying it.

- Increase wait time by a few seconds.
- Allow the student more tries before you recast or reword what they have said.
- Ask clarifying questions:

Can you tell me a bit more?
Can you explain it again?
Can you expand on that a little more?
What do you mean?

Finally—and most importantly—the teacher responds to the meaning of what the student has reported. The teacher listens to what the student says and recasts the student's report to summarize.

Which is farthest? Explain in words & numbers

Meadow View School sixth-grade teacher Josh Deis, above, used Lego robots and directed students to "talk to a neighbor" to bring this math lesson alive. Students worked in groups to measure the distances their robots traveled, then compared the distances to the total length of the track. They converted their measurements to fractions, decimals, and percentages and discussed the concept of "part of a whole" with each other.

Types of guiding questions

Refocusing questions are used to help students initiate thinking and focus on a topic.
What makes you say____?

Clarifying questions focus student thinking and encourage them to develop more precise language.
What do you mean by ____?
Can you say that in different words?

Verifying questions encourage students to cite evidence that supports their thinking.
Give me an example of ____.
How do you know that?

Redirecting questions broaden student thinking and expand the dialogue.
What else do you (recall, observe, infer) about ___?

"Narrowing the focus" questions limit discourse to topics of key importance.
Can you tell me more about ____?

Supporting questions help students make connections between evidence and conclusions.
Why do you think ____ will result in ____?

This publication was developed by the Sonoma County Office of Education in support of Aiming High. For information, contact Jane Escobedo, (707) 522-3305.

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